Class on Engle-Granger and VARs

Exercise 7 from the Homework

You were asked to take two time series and check if they were cointegrated.

First, run OLS of Investment over Exports (let's add a constant in the right hand side). Notice that:

1) The R^2 is very high (greater than the Durbin Watson)
2) The t and F statistics are highly significant.

Let's check the residuals. We will have to see if they're stationary or not.
Proc/Make residual series

A quick look at the line graph of the residuals shows clearly that they are not stationary. Let's test this proposition more formally.
View/Unit root test

Cointegration tests are generally unit root tests applied to residuals. There are several cointegration tests. Some postulate the null of a unit root in the residuals/null of no cointegration (Cointegration Regression Durbin Watson; Augmented Dickey-Fuller; Phillips-Ouliaris); others postulate the null of no unit root in the residuals/null of cointegration (Shin, who draws on KPSS; Leybourne and McCabe). We will see here only the ADF and Phillips-Ouliaris (the Durbin Watson has been proven less reliable for the study of cointegration by Engle and Yoo(1987)).[1]

You could use the *Phillips-Ouliaris test*, which amounts to apply the Phillips-Perron test to the residuals. Here, whatever the form we give to the mean process, we do not add trend nor a constant in the residuals regression.

The test statistic will be –1.76. Eviews displays Mackinnon's critical values, which are good since they are adjusted to the sample size. The ADF t test and the Zt test have the same asymptotic distribution; hence, their critical values should asymptotically coincide. However, if you want to use the Mackinnon (1991,1996) tables, given by Eviews, you should not include either a trend or a constant in the mean process.
Let's take a look at the Phillips-Ouliaris tables (you will find them in Hamilton). Ours is case 2. At the 5% level, the asymptotical critical value is –3.37. Hence, we do not reject the null of nonstationarity.

[1] You can find an extensive list of these tests in Maddala and Kim textbook and in Robalo textbook.

You could also choose the _Cointegration ADF_, for instance (remember: here, lags will be added to the right hand side of the Dickey Fuller regression)
We would like to use Mackinnon's critical values. To that end, we regress investment over exports without a constant and put a constant in the residuals. The critical values given by Eviews should thus be correct.
Conclusion: t-stat is –0.505. Mackinnon's tables for the cointegration test ADF (1991,1996) are useful because they account for small samples, such as ours.[2] But the table's critical values were derived without putting a constant or a trend in the mean. The form for the residuals process is free.[3]

This means the regression is spurious and there is no cointegration between the variables.

Note: in large samples, the results of estimating the regression the other way around, should be compatible with the results derived under the normalization assumed here.

Engle-Granger methodology (example taken from Enders, chapter 6)

Definition: the components of the vector $x_t = (x_{1t}, x_{2t}, ..., x_{nt})'$ are said to be cointegrated of order d, b, denoted by $x_t \sim CI(d,b)$ if:

1. **All components of x_t are integrated of order d.**
2. **There exists a vector $\beta = (\beta_1, \beta_2, ..., \beta_n)$ such that linear combination $\beta x_t = \beta_1 x_{1t} + \beta_2 x_{2t} + ... + \beta_n x_{nt}$ is integrated of order (d-b), where b>0.**

I will pick an example from Enders, chapter 6, where he simulates three series: y, z and w. The Engle-Granger approach suggests the following steps:

1) *Pretest the variables for their order of integration.* For instance, if you have two variables, where one is I(1) and another is I(2), they are not cointegrated. See the previous exercise. Investment is I(1); but exports are probably I(2). They could never be cointegrated.
2) *Run OLS on the potential cointegration relation.* Run the Phillips Perron test over the residuals. It is –5.35. Table B.9 - case 2 for the Phillips-Ouliaris test gives us a critical value of **–3.37**. Hence, we reject the null of no cointegration (you could additionally use ADF, Shin, etc…).
3) *Generate the residuals from the previous step and plug them into the Error correction mechanism specification*. Note: adding a constant in the error mechanism process and outside it has different implications for the behaviour of the series.

The Error Correction Model has the form

$$\Delta w_t = \gamma_0 + \alpha e_{t-1} + \gamma_1 \Delta y_{t-1} + \gamma_2 \Delta z_{t-1} + \gamma_3 \Delta w_{t-1} + \varepsilon_{wt}$$

[2] See e.g. Banerjee et al textbook or Robalo.
[3] See e.g. Banerjee et al textbook or Robalo on the equivalence of both approaches.

where $e_{wt-1} = w_{t-1} - \beta_0 - \beta_1 y_{t-1} - \beta_2 z_{t-1}$ are the estimated residuals from the estimated cointegration relation. The parameter α denotes the speed of adjustment and it should be negative to be meaningful from an equilibrium point of view: the intuition is that, if the residual is positive, it will decrease to its equilibrium level, since it was too large.
This is known as the two step approach by Engle and Granger

4) *Assess model adequacy*. We can see if the residuals approximate a white noise by looking at the correlogram and at the Ljung Box statistic; they do.
Finally, we can run tests about the cointegrating vector, as explained in pages 610 and 611 in Hamilton. If the regressor in the "equilibrium" relation is uncorrelated with the residual, we can use the traditional t-stat. In the event of correlation between residual and regressor, we should correct the test statistics
Similarly to an ADF test, we may add leads e lags to the right hand side, a procedure suggested, among others, by Stock and Watson(1993). This would be a huge task with three variables. The correction will use the estimated variance of the residual of the "equilibrium" regression (or Newey West if there is autocorrelation) and the estimated variance of the regression with the leads and lags.

Just to show this test as an example, we took only two (obviously cointegrated) variables, z and z2. And we added lags and leads to the right hand side variable (taken to be z). Then we corrected the t-stat, using the standard deviation of the residuals and the coefficients of an estimated autoregressive process in the estimated residuals.
Since the variable z2 was randomly generated in class, I will not reproduce the results here. Instead, I prefer to give you an idea on how to perform this test for two variables, consumption c and disposable income y. You can see Hamilton (chapter 19) on this.

First, we add lead and lags to the right hand side and estimate

$$c_t = \alpha_0 + \alpha_1 y_t + \alpha_2 \Delta y_{t+2} + \alpha_3 \Delta y_{t+1} + \alpha_4 \Delta y_t + \alpha_5 \Delta y_{t-1} + \alpha_6 \Delta y_{t-2} + \hat{v}_t$$

Then, $\hat{\sigma}_v^2 = \dfrac{\sum \hat{v}_t^2}{T-7}$

Second, to test if the coefficient of y is equal to one, we should write the tstat as $t = \left(\hat{\alpha}_1 - 1\right) / stdev$.

Now, adjust an AR(k) to the residuals of the regression and compute $\hat{\sigma} = \dfrac{\hat{\sigma}_e}{1 - \hat{\phi}_1 - \hat{\phi}_2 - ... - \hat{\phi}_k}$ given that $\hat{\sigma}_e^2$ is the variance of the residuals of the AR(k) on the residuals, which could alternatively be obtained through the Newey-West estimator.

Finally, we can compute the test statistic as $t\left(\frac{\hat{\sigma}_v}{\hat{\sigma}}\right)$ which we should compare with the N(0,1) critical value of 1.96.

Reduced form VARS and structural vars

Today we will work still mainly with reduced form VAR's. Open Warne's SVAR package. Choose the data that come from Bruggeman, Donati and Warne(2003) about money demand in the eurozone.
As for the model, let's begin with a simple one: a VAR(2) (in general, we must choose the lag structure so that correlation is wiped out from the residuals). Go to Parameters and choose a non-structural VAR without cointegration.
Also go to **Tools/Preferences/Output control/Write Granger causality… and Open output file directly after estimation**

Let's run the model by clicking the *hammer icon* on the toolbar.

Definition: we say that a variable x Granger causes y if it helps to predict y. We test this by applying an F test to all lags of x in y's equation.

What do the results tell us about Granger causality? For instance, what variables Granger cause the money supply?
The null is of no Granger causality; it is basically an F test asymptotically following a chi-square distribution with m degrees of freedom (m being the autoregressive lag length). *As explained in Hamilton, page 304, the point is the following: if the sum of squared residuals in y's equation changes substantially (downwards) when we include past lags of x, we tend to reject the null of no Granger causality.*

We conclude, for instance, that output y and the long run interest rate il Granger cause money (we reject the null).

To estimate a structural var, choose Structural Var in "Model Type Selection". We have 6 endogenous variables. This means we will need to have 6(6-1)/2 restrictions for an exact identification.

In order to do forecasting, choose the rocket icon on the toolbar. You may choose between within sample and out of the sample forecasting. You will get 6 graphs with forecasts and confidence intervals just as many as the number of endogenous variables you have in the model.

Try using 14 restrictions just to see what SVAR tells you.

If you impose more than the 15 restrictions, you will have an overidentified model: it is possible, by using a chi-square test, to test for the significance of these restrictions (see Enders, chapter 5).