Tax and Public Expenditure Policies under Government Constraints

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Abstract

The extensive literature on optimal taxation makes two unrealistic assumptions: resources used by the government are "dumped into the river" and initial capital may be expropriated. This paper jointly evaluates the effects of including the channels of influence of public expenditures (public goods increase utility, physical infrastructure rises private capital productivity and human capital contributes to labor productivity) and consumption and factor taxes. We also deal with the issue of dynamic inconsistency, and show that there is a tradeoff between consistency and capital taxation. We compute both Ramsey optimization and steady-state welfare maximization in a balanced budget context. We show the optimality of consumption taxation, and, if taxes on consumption are administratively restricted, then the rule that capital should not be taxed does not hold. Moreover, in this case, the design of an optimal public policy requires the simultaneous definition of expenditure and tax structures. Transition paths are also studied.
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1 Introduction

There is already an extensive literature on the optimal capital tax rate in an intertemporal model. Recent papers include Chamley (1981), Judd (1985), Chamley (1986), Lucas (1990), Jones, Manuelli and Rossi (1993), Judd (1999) and Cole.

Most of these theories lead to a somewhat unrealistic conclusion, that a dynamic optimal tax policy should expropriate capital revenues at the beginning of time, followed afterwards by a zero or a low rate of taxes on capital. Lucas and Cole provide a clear intuition for this result.

However, we do not observe such behaviour in the real world. First, if the state carries out a once for all expropriation, it should translate into a high budget surplus at the beginning of time. Over long periods of time, most of the deviations from budget balance take the other sign: they are large build-ups of large public debt. And empirical studies show that this large build-up of government debt is due to war efforts or large build-up of public infrastructures. Second, there is the problem of intertemporal consistency. And third, at the beginning of any growing economy, with an infinite period, consumption and GDP per capita must be low compared with levels at the end of time. Under the Wagner law, there are several reasons why taxation levels at the beginning of time should be low, when compared with the end of time, even on relative terms. Exactly the contrary the result obtained in the literature.

In order to obtain results that are more realistic, we have to impose more structure on the problem. First, we set a government balanced budget on every period, an assumption that does not seem unreasonable in a long-term growth model with no economic fluctuations or unemployment. Also, this assumption enormously simplifies the numerical solution of the model. Another assumption that is central to build a realistic fiscal policy model is the introduction of consumption taxes, that are today widely used in some form. Moreover, a more realistic model should consider three major uses for government resources: production of public goods raises the representative agent's utility, physical infrastructure raises private capital productivity, and human capital contributes to labor productivity. When public expenditure enters the utility function as a substitute for private consumption, or has some impact on the production of investment goods, governments can claim a positive share of resources over the long term. Barro (1990), Barro and Sala-I-Martin (1992) and Jones et al. (1993, model 3) find that, even in such a case, optimal tax rates are very low when compared with real-world values.

Our paper is thus an extension of Coleman in two regards. First, it introduces balanced budget in the optimal Ramsey model. Second, it adds the hypothesis that government expenditure can either be productive or generate directly utility to the people.

Even in the presence of budgetary constraints, optimal second-best policies using a Ramsey criterion are still intertemporally inconsistent. Governments can, at any time, renege on previous engagements and recalculate another optimal policy. Although it is very difficult to find rules that are consistent in generalized settings, the balanced budget rule is one of the simplest that can be found. The issue of the dynamic inconsistency of optimal fiscal and budgetary policies leads to the quest for other welfare maximization criteria that are consistent in the sense that the prescribed optimal policies (i) do not depend on the moment of optimization, and (ii) indeed contribute to an increase in welfare. As noted by Turnovsky and Brock (1980), the maximization of steady-state welfare is consistent ??? *TRIVIAL*. We thus compare some theoretical features of taxation and spending under both criteria: (i) Ramsey optimization and (ii) steady-state welfare maximization. One conclusion of this exercise is that consumption taxation is better than factor income taxation and, contrary to Jones et al.'s (1993)results under fiscal imbalances, should never be zero asymptotically. We also show that, if consumption taxation is exogenous or bounded from above (for political or practical reasons), the rule that capital should not be taxed does not hold. FOLLOWING THE WORK OF COLEMAN WE RESTRICT TAX RATES TO BE NON-NEGATIVE. Finally, quantitative results suggest that there might be a trade off between time-consistency and low tax rates on factor income.

We also investigate the effects of fiscal constraints on the design of fiscal policy. If consumption taxes cannot entirely finance public spending, the design of an optimal public policy requires the simultaneous definition of expenditure and tax structures. *These two aspects of any reform are intermingled,* exploited in a companion article.

The next section sets up the theoretical model used throughout and characterizes the behavior of the economy's steady state. In the following section we discuss two alternative criteria for welfare maximization. Next, we study a stripped-down version of the model to introduce a simple taxation benefit principle. We then turn to some theoretical results on expenditure and fiscal policies under two optimization regimes. Finally, we provide a numerical example that tries to illustrate the main aspects of the previous sections. The transition path of the economy is also computed. The last section concludes.

2 The model

We shall not carry out an extensive treatment of social equality issues typical from Mirrlees's (1971) optimal taxation theory. What are the main requisites for this model? In order to study tax reforms, the government must be able to raise taxes from at least two sources: income on work and capital. Consumption taxation will also be considered. For the purpose of analyzing public expenditure rules, public spending on infrastructures, for the one side, and education and health, for the other, will enhance physical and human capital productivity, respectively. We now proceed to a mathematical description of the model.

2.1 The households

The economy is populated by a large number of identical households. The intertemporal utility of the typical working household is

$$\int_0^\infty U(c_t, g_{c,t}, l_t)e^{-\rho t}dt, \tag{1}$$

where

$$U(c, g_c, l) = \frac{(c^\theta g_c^{1-\theta} l^\varepsilon)^{1-\sigma}}{1-\sigma}dt.$$

Here, ρ is the rate of time preference. c_t and $g_{c,t}$ are per capita consumption and government spending in administration at time t, respectively, and l_t is the amount of available time (normalized to E units) that the agent dedicates to leisure activities.[1] ε is the elasticity of leisure. Positive parameter θ captures the importance of private consumption, c, relative to government spending on administration, g_c. Parameter σ is the inverse of intertemporal elasticity of substitution.

SHOULD WE ASSUME THAT $\lim_{t\to 0} g_c = 0 (Wagnerlaw)$??? I think this will engogenously happen provided we start with a low capital endowment. We could think of eliminating this altogether.

The households are initially endowed with given amounts of physical and human capital, k_0 and h_0. At time t, they own quantities k_t and h_t and invest amount i_t and $i_{h,t}$ in physical and human capital, respectively, in order to collect payments from their stock of capital and working time, which we shall assume to be competitively determined. Many governments use tax revenue in part to invest and purchase goods and services, and in part to transfer resources among households. We shall assume that there is an exogenous stream of such transfers to households, b_t. Their resources constraint is then

$$c_t\left(1+\tau_{c,t}\right) + i_t + i_{h,t} \le r_t k_t + w_t h_t u_t + b_t, \tag{2}$$

where r_t is after-tax return on capital, w_t is after-tax wage rate, u_t is the amount of working time, and $\tau_{c,t}$ corresponds to a flat-rate tax on consumption.

The stock of capital is subject to the motion equation

$$\dot{k}_t = G\left(i_t, g_{k,t}\right) - \delta k_t, \tag{3}$$

[1] Rather than being an usual public good, g_c is best described as a publicly provided private good. If it were a pure public good, the total amount of public administrative expenditures would be the relevant variable to be placed in the instantaneous utility function. This model would then depend on the scale of the economy, an issue that we will not address here. Barro and Sala-I-Martin (1995) argue that very few public goods are indeed nonrival and nonexcludable. They prefer a formulation where public activities are subject to congestion, which amounts to saying that one may use publicly provided goods on a per capita basis.

where G is a twice differentiable, strictly increasing and strictly concave in all arguments, homogeneous of degree 1 function, and $g_{k,t}$ is the amount of government investment in physical capital. The stock of human capital is driven by equation

$$\dot{h}_t = H\left(h_t, i_{h,t}, g_{h,t}\right) - \chi h_t, \tag{4}$$

where H is a twice differentiable, strictly increasing and strictly concave in all arguments, homogeneous of degree 1 function, and $i_{h,t}$ and $g_{h,t}$ are private and public investment in human capital, respectively.[2] δ and χ are depreciation rates for the capital stocks. We shall adopt a Cobb-Douglas shape for functions G and H. Working households seek to maximize (1) subject to constraints (**??**), (**??**) and (**??**). They control trajectory $\{c_t, i_t, i_{h,t}, u_t, l_t\}$ and take as given path $\{r_t, w_t, g_{c,t}, g_{k,t}, g_{h,t}, b_t, \tau_{c,t}\}$. We shall abstract from scale considerations in this model, where it is assumed that population remains constant.

2.1.1 First order conditions

By setting up an appropriate Lagrangian function,

$$\begin{aligned} \mathcal{L} = &\, U(c, g_c, l) + \psi\left(rk + whu - c\left(1+\tau_c\right) - i - i_h\right) \\ &+ \lambda_1\left(G(i, g_k) - \delta\, k\right) + \lambda_2\left(H(h, i_h, g_h) - \chi h\right), \end{aligned} \tag{5}$$

it can be shown that Pontryagin's Maximum Principle implies two main conditions for the working household:

$$\frac{e^{-\rho t} U_c\left(t\right)}{1+\tau_{c,t}} = \frac{e^{-\rho \xi} U_c(\xi)}{1+\tau_{c,\xi}} R_{t,\xi}, \tag{6}$$

where

$$R_{t,\xi} = \frac{G_i(t)}{G_i(\xi)} \exp\left\{-\int_{\xi}^{t} \left(r_u G_i(u) - \delta\right) du\right\}.$$

The first condition says that the marginal rate of substitution of consumption at two different moments equals the relative net price of consumption at those two moments. Note also that consumption's intertemporal marginal rate of substitution is the same whether it is calculated using the net return of investment in physical or human capital, which implies

$$R_{t,\xi} = \frac{H_{i_h}(t)}{H_{i_h}(\xi)} \exp\left\{-\int_{\xi}^{t} \left(H_h(u) + w_u H_{i_h}(u) - \chi\right) du\right\}. \tag{7}$$

This is the usual efficiency condition in the allocation of factors.

The second condition is

$$\frac{U_l\left(t\right)}{U_c\left(t\right)} = \frac{w_t h_t}{1+\tau_{c,t}}. \tag{8}$$

This equation states that, at the margin, the agent must be indifferent between engaging in leisure activities and working.

As in the theoretical framework discussed in Lucas (1990), equation (6) retains its strict interpretation that the marginal rate of substitution between consumption at times t and ξ must equalize the net relative prices of these two goods. In a more general specification, however, we can see that the time discount factor includes the marginal productivity of private investment in accumulating physical capital, G_i. Since function G depends on both private and public investment, this means the government exerts an active and presumably important role in the determination of equilibrium. A similar reasoning could be made about function H.

[2]Throughout this paper, if $F(x, y)$ is an appropriate function, F_x denotes that function's partial derivative with respect to variable x. Of course, F_{xy} is the function's second derivative with respect to x and y. Occasionally, we may use the 'D' or the Leibniz notation, in which case, for instance,$F_x = D_1(F) = \frac{\partial F}{\partial x}$ and $F_{xy} = D_{12}(F) = \frac{\partial^2 F}{\partial x \partial y}$. For simplicity, we will generally omit the function's arguments evaluated at time t, writing instead $F(t)$.

2.2 Production

The production side of this economy is composed of an infinite number of identical firms, each possessing a constant returns to scale technology. The output depends solely on the existing stock of physical capital, k, and effective hours worked, $n = hu$. The production function, $F(k, n)$, is assumed to be twice continuously differentiable, strictly increasing and strictly concave in both arguments. Competition between firms leads to the familiar conditions

$$(1 - \tau_{k,t}) F_k(t) = r_t \tag{9}$$

$$(1 - \tau_{w,t}) F_n(t) = w_t, \tag{10}$$

where $\tau_{w,t}$ and $\tau_{k,t}$ are flat-rate taxes the government imposes on work and capital income, respectively. Again, we shall assume a Cobb-Douglas functional form for F.

2.3 Balanced budget

We will not allow bonds so that the primary deficit is always zero. Therefore, by "balanced budget" we mean a balanced primary budget. As with Klein, Quadrini and Ríos-Rull (2000), one reason for this is that the numerical procedures used to solve the model turn out to be much simpler. We can therefore focus more specifically on a long-term equilibrium under different optimization criteria without having to worry about the very difficult problem of setting up an optimal policy with surpluses and deficits.

This "balanced budget" hypothesis seems relevant in a long-term approach. The deficit instrument (operated by emitting/paying debt, lowering/raising taxes, or any other means) must, in our opinion, be viewed as temporary. If one is interested in a long-run analysis, it does not make much sense to admit the possibility of an asymptotic deficit or surplus. In a shorter term, any optimal policy involving temporary deficits or surpluses is time-inconsistent.[3] Recent contributions on this issue seem to indicate that, contrary to the results of Chamley (1986) and Judd (1999), among others, which advocate near-zero rates on capital income, the observed tax rates are relatively high due to the lack of an effective technology of commitment for the government. Klein and Ríos-Rull (2000) propose a stochastic growth model in which the government can only commit for one period in one of the fiscal instruments (the tax rate on capital), whereas the other (the labor tax rate) adjusts to balance the budget. In this setting, they obtain non-zero tax rates on capital income, as well as the usual result that work income should indeed be taxed. Their results therefore suggest that time inconsistency may play an important role in the design of an optimal fiscal policy. Stockman (2000) finds that high tax rates may be due to politically motivated restrictions on the level of public debt or fiscal deficit.

The hypothesis of a balanced fiscal budget is also consistent with policy rules popularized by many governments in recent years, especially in Europe. This is a particularly interesting case, since there have actually been imposed near-zero budget rules over the European Union (EU) members. Representatives of the EU members set up these rules since it is deemed a "good thing" for the EU as a whole. One reason for this might be the need to ensure that no member state will indulge in unrealistic, over-spending policies that will render the Euro weaker by well-known effects. Dixit (2001) justifies the imposition of rules such as these by arguing that freedom of national fiscal policies undermines the European Central Bank's monetary commitment. Since there are enforceable high penalties for those who break the rules (in the absence of which breaking the rules would be in everybody's best private interest) it is in no one's interest to spend too much. These political issues are analyzed by Alesina (1988) and Corsetti and Roubini (1992), for example, and lie much beyond the scope of this text.

We assume that part of the government's revenue is invested in both physical (g_k) and human capital (g_h), part is spent on administrative actions (g_c), and part is transferred to the non-working part of the population ($(1 + \tau_{c,t}) b_t$). It is easily seen that the government has a constraint of the form

$$\tau_{k,t} F_k(t) k_t + \tau_{w,t} F_n(t) h_t u_t + \tau_{c,t} (c_t + b_t) = g_{c,t} + g_{k,t} + g_{h,t} + (1 + \tau_{c,t}) b_t. \tag{11}$$

It can be shown that equality (11) and the resources constraint (12) below together imply that inequality (**??**) be just satisfied.

[3] Debt issues will not be addressed here. Since the "balanced budget" assumption is essentially the same as imposing a fixed deficit as a fraction of output, the assumption we shall make will be that debt b evolves according to $\gamma_b = \frac{y}{b} \Delta_{\mathrm{PD}} + r$, where r is the net interest rate on bonds and Δ_{PD} is the exogenous primary deficit as a fraction of output.

NOTE: STEADY STATE ANALYSIS IS NOT INTERESTING FOR A GROWING ECONOMY, UNLESS WE ALSO INVESTIGATE THE TRANSITION PATH. THIS IS BECAUSE TAX RATES MAY -AND THEU USELY DO- CHANGE OVER THE GROWTH PATH (E.G. WAGNER LAW).

[*** We shall be interested in the transition path. In order to compare this model with Coleman's, we shall use another approach — take g_t as given and proceed with exactly the same calibrations. I am sure we shall get much more realistic figures.

2.4 The steady state

In order to reduce the dimensionality of the optimization program, we can eliminate prices r and w by using equations (9) and (10). On the other hand, it is necessary that in equilibrium the resources constraint holds:

$$F(t) = c_t + g_t + i_t + i_{h,t} + b_t, \tag{12}$$

where $g_t = g_{c,t} + g_{k,t} + g_{h,t}$.

We now proceed to the definition of the steady-state equilibrium. Denote by γ the economy's endogenously determined growth rate. γ is strictly positive only if functions G and H are homogeneous of degree one. By convention, a variable named x will represent the ratio x_t/y_t in the steady state, where $y_t = F(t)$. A *steady state equilibrium* is an allocation which conforms to the system

$$\gamma k = G(i, g_k) - \delta k \tag{13}$$
$$\gamma h = H(h, i_h, g_h) - \chi h \tag{14}$$
$$1 = c + g + i + i_h + b \tag{15}$$
$$g + b = \tau_w h F_n(k, hu) + \tau_k k F_k(k, hu) + \tau_c c \tag{16}$$
$$\rho + \gamma\sigma + \delta = (1-\tau_k) F_k(k, hu)\, G_i(i, g_k) \tag{17}$$
$$\rho + \gamma\sigma + \chi = (1-\tau_w) F_n(k, hu)\, H_{i_h}(h, i_h, g_h) u + H_h(h, i_h, g_h) \tag{18}$$
$$\frac{\varepsilon c}{\theta l} = \frac{(1-\tau_w) F_n(k, hu)\, h}{1+\tau_c} \tag{19}$$
$$E = u + l \tag{20}$$
$$1 = F(k, hu). \tag{21}$$

As in Jones et al. (1993), the hypothesis that the economy indeed reaches a steady state is not innocuous. We shall thus focus on policies that are feasible, stationary and lead to some steady state. Is there anything we can say about the relative merits of each equilibrium? The answer to this involves optimizing the social welfare function, which takes us to the realm of second-best allocations.

NOTE:BE SPECIFIC IF TAX RATES ARE CONSTANT OVER TIME, OR IF THEY CAN VARY OVER TIME? [on the ss, they're constant. During the transition path, they vary

QUESTION: DOES A CONSTANT CONSUMPTION TAX RATE GENERATE EFFICIENCY GAINS SIMILAR TO A RAMSEY POLICY WHERE TAX RATES VARY OVER TIME AS IN COLEMAN?

[I don't think so.

3 The Ramsey problem

If the government is a benevolent one and wants to maximize the household's lifetime utility, the obvious way to proceed is to maximize lifetime utility taking as given the household's decision rules. The government can commit itself to some policy trajectory, which defines the amount of public expenditure on a number of items, as well as tax rates on physical or human capital, conditional on household's decisions. Based on this path, households solve their own maximization problem. The solution to this sort of problem has been widely characterized by several authors (see, for instance, Arrow and Kurz (1971), Chamley (1986), Lucas (1990), Jones et al. (1993), Judd (1999) or Turnovsky (1995)). Usually, it involves initially taxing physical capital at confiscatory levels, and later reducing those taxes. This result is hardly surprising since capital is relatively inelastic in the short run and capital taxes are distortionary. Therefore, the government wants to reduce the deleterious dynamic effect of taxes, which it does by lowering tax rates on capital as the economy adjusts. This procedure is often seen as a mere theoretical benchmark since no government is able to carry out such an unpopular policy for political as well as practical reasons.

The proposition that steady-state capital taxation must be zero or close to zero has thus obtained wide acceptance, even if policymakers have not adopted it. Capital taxation is usually much heavier than it should be if the theory were taken into account. In an attempt to explain this observation, Stockman (2000) uses politically motivated budget rules in a model with public debt to allow for high optimal tax rates.

Another problem with optimal plans under full commitment is time inconsistency. The seminal work by Fischer (1980) alerted economists to a problem that is both difficult and important. Several methods for alleviating the fact that, if the government lacks a commitment technology, the outcome of its optimization problem depends on the moment where it decides to solve it, have been proposed. (In Section 2.3, some methods for avoiding this problem are mentioned.) Klein and Ríos-Rull (2000) suggests that the lack of a commitment technology may help explain why taxes are in practice so much higher than the theory predicts. Turnovsky and Brock (1980) point out some cases where optimal plans are time-consistent.

The Ramsey problem that we intend to characterize could be phrased as: "find the best trajectories for all variables—including tax rates—such that the weighted intertemporal utility of both types of households is maximized, all technical restrictions are satisfied, and the outcome is compatible with a competitive equilibrium". More formally, the government wants to maximize

$$\int_0^\infty \bar{U}(c_t, g_{c,t}, l_t, b_t) e^{-\rho t} dt, \tag{22}$$

where

$$\bar{U}(c, g_c, l, b) = \frac{(c^\theta g_c^{1-\theta} l^\varepsilon)^{1-\sigma}}{1-\sigma} + \frac{s}{1-s}\frac{b^{1-\sigma}}{1-\sigma}, \tag{23}$$

with respect to trajectory

$$\{c_t, i_t, i_{h,t}, u_t, l_t, g_{k,t}, g_{h,t}, g_{c,t}, b_t, k_t, h_t, \tau_{k,t}, \tau_{w,t}, \tau_{c,t}\},$$

subject to constraints (**??**), (**??**), (6), (8), (7), (11) and (12) given k_0 and h_0, with r_t and w_t eliminated by use of equations (9) and (10). The formulation of such a problem is by no means an easy exercise, and involves the creation of auxiliary variables that account for restriction (6). Also, we should note that this is not a first-best equilibrium since the government is not entitled to directly allocate resources to the private sector.

Due to the imposition of balanced budget at all times, for a given policy this model's steady state may be characterized without calculating the transition path. As long as we do not calculate the transition path for the Ramsey problem, we cannot assess the level of each growing variable at a particular moment; but we are able to obtain the limiting ratios to one of them (say, output y or human capital h). We will thus work with shares of output rather than levels. As usual in this sort of model, this is only possible because of the homogeneity of the objective function's partial derivatives, the homogeneity of degree one of the motion functions, and the homogeneity of the constraints. If there were intertemporal constraints expressed in present value form that must be verified initially (as in Jones et al. (1993)), this result would not generally hold and the transition path would have to be calculated. We now study what happens in this model under different hypotheses: lifetime or steady-state utility maximization; exogenous or optimal consumption tax rates. The exogeneity of the consumption tax rate includes the familiar case where *there are no consumption taxes.*

3.1 The steady-state allocation

Let us first proceed to the case where taxes on consumption can be freely chosen. The situation where taxes on consumption cannot be adjusted is then approached. This latter hypothesis encompasses a setting in which there are no consumption taxes at all, a feature of many models of this type.

3.1.1 Adjustable tax rate on consumption

The classic result from the optimal fiscal policy literature that steady-state capital taxation should be zero still holds if we have enough fiscal instruments at our disposal. However, in our model, due to the presence of consumption taxes, taxes on labor revenue must also vanish.

Proposition 1 *There is one steady-state equilibrium allocation $P \in S$ such that intertemporal utility (22) is maximized; tax rates on work and capital are zero; and the consumption tax rate is set to just cover total government expenditure. The structure of spending is found by equating the marginal benefit of an additional unit of public spending in a given sector to its private counterpart.*

(See the proofs of this and the remaining propositions in the appendix.) The previous allocation is interesting in that it confirms (i) that consumption taxing is optimal in the long run and should be carried out to its maximum possible extent; and (ii) that public spending on education, infrastructure and services should be set looking at private decisions. The balanced budget constraint eliminates the possibility that all tax rates are simultaneously zero. This contrasts with the results of Jones, Manuelli and Rossi (1997), which state that even the consumption tax rate should basically be zero in the long run for a wide class of utility functions. Therefore, consumption taxation is generally preferable to taxation over accumulable factors.

Unfortunately, there are several drawbacks to this policy. The first is pointed out in the following proposition.

Proposition 2 *The Ramsey problem is always dynamically inconsistent in that, despite the fact that the primary budget is satisfied at all times, it yields a time-dependent solution.*

Every time the government performs this optimization problem, it has an incentive to tax intermediate goods (as are both types of capital) heavily. This plan can be implemented only if there is a suitable commitment technology, which is by no means an innocuous supposition.

The Ramsey solution in this model yields essentially the same results as in the literature regarding capital income taxation, which state that taxes on capital must asymptotically be zero. As Jones et al. (1997) point out for the case where working skills can be accumulated, there is nothing special about taxes on work, and they must also be zero in the long run. Contrary to these authors, however, taxes on consumption must be positive because no fiscal deficits are allowed. This fact illustrates the important conclusion that *political (or practical) restrictions on fiscal policy have a strong impact on the optimal tax structure and, therefore, on the outcome of the economy.*

3.1.2 Exogenous tax rate on consumption

Numerical simulations for "realistic" parameters, to be presented below, show that steady-state consumption taxes must be very high in the steady state. It then seems plausible to admit that, due to practical (political, institutional) reasons, the government cannot finance all its needs by the consumption tax. Thus, asymptotic tax rates on income should not be zero.

Suppose now that $H_{i_h}(h, i_h, g_h) = 0$, that is, human capital accumulation only depends on public investment and existing human capital. This is, for instance, the case where all private input is exogenously supplied through learning time. Then, the following proposition holds.

Proposition 3 *If $H_{i_h}(h, i_h, g_h) = 0$, then steady-state taxation on physical capital is zero and all tax revenue comes from labor and consumption taxes.*

This result could help explain why countries with high public spending on education (e.g. the Scandinavian countries) tend to have high taxes on wages when compared with countries like the United States or UK. Proposition 3 still holds in the case where consumption taxes adjust (in which case they are zero), provided that the labor share of output is higher than the efficient public expenditure.

4 Conclusions

This paper has implemented a simple growth model of intertemporal equilibrium incorporating public consumption in the social welfare function of a representative agent, as well as the impact of physical infrastructures and human capital within an endogenous growth model. We have assumed throughout a balanced budget. This model does not include an explicit treatment of social security payments. We split the analysis of government's behavior into two cases: Ramsey optimization problem and steady-state welfare maximization. In our formulation, the endogenous variables gradually approach the steady state. The relative magnitude of steady-state variables does not depend on the transition path.

One major theoretical finding is that, under a Ramsey framework and with a sufficiently flexible tax system, the optimal steady-state tax structure involves setting zero tax rates on factor income, with all public spending being financed through consumption taxes, even in the presence of productive public spending and a balanced fiscal budget at all times. This result illustrates the strong impact of fiscal constraints on the design of optimal fiscal policy. If consumption taxation is constrained either because practical, institutional or political reasons, or because consumption taxation is exogenously given or nonexistent, the optimal tax rates on factor income are not zero in the steady state. Quantitatively, taxes on physical capital tend to be small compared to taxes on human capital.

The picture changes when the Ramsey framework is replaced by steady-state welfare maximization (SSWM), a time-consistent criterion. Here, not only are asymptotic tax rates not equal to zero, but they can also be quite surprising. With a fully adjustable consumption tax rate, quantitative results—robust to a wide variation of parameters in an actual economy—suggest that, at the optimum, there might actually exist a negative tax on work and a high tax rate on physical capital. With an exogenously given tax rate on consumption, numerical results fairly well mimic actual, observed values. In that case, a switch to an inconsistent policy such as Ramsey's would not imply a considerably higher steady-state growth rate for the economy as a whole.

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A Proofs

A.1 Proposition 1

Proof. We can solve the government's problem by setting up the Hamiltonian expression:

$$\begin{aligned}
\mathcal{H} = {} & U(c, g_c) + \eta_1 \left(\frac{\partial}{\partial c} U(c, g_c) + \psi \left(-1 - \tau_c\right) \right) + \\
& \eta_2 \left(\lambda_2 D_2(H)(h, i_h, g_h) - \psi\right) + \eta_3 \left(\lambda_1 D_1(G)(i, g_k) - \psi\right) + \\
& \eta_4 (D_1(F)(k, h) \left(1 - \tau_k\right) k + \\
& D_2(F)(k, h) \left(1 - \tau_w\right) h - \left(1 + \tau_c\right) c - i_h - i) + \\
& \eta_5 (\tau_c\, c + \tau_k\, D_1(F)(k, h) k + \\
& \tau_w\, D_2(F)(k, h) h - g_h - g_k - g_c) + \\
& \mu_1 \left(\lambda_1 \delta - \psi D_1(F)(k, h) \left(1 - \tau_k\right) + \rho \lambda_1\right) + \\
& \mu_2 (-\lambda_2 \left(D_1(H)(h, i_h, g_h) - \chi\right) + \\
& - \psi D_2(F)(k, h) \left(1 - \tau_w\right) + \rho \lambda_2) + \\
& \mu_3 \left(G(i, g_k) - \delta\, k\right) + \mu_4 \left(H(h, i_h, g_h) - \chi h\right),
\end{aligned}$$

where constraints come from the household's optimization problem and policy feasibility. The first order conditions are:

$$\frac{\partial \mathcal{H}}{\partial c}=0;\ \frac{\partial \mathcal{H}}{\partial i}=0;\ \frac{\partial \mathcal{H}}{\partial i_h}=0;\ \frac{\partial \mathcal{H}}{\partial \psi}=0$$
$$\frac{\partial \mathcal{H}}{\partial g_c}=0;\ \frac{\partial \mathcal{H}}{\partial g_k}=0;\ \frac{\partial \mathcal{H}}{\partial g_h}=0$$
$$\frac{\partial \mathcal{H}}{\partial \tau_c}=0;\ \frac{\partial \mathcal{H}}{\partial \tau_k}=0;\ \frac{\partial \mathcal{H}}{\partial \tau_h}=0$$
$$\frac{\partial \mathcal{H}}{\partial k}=-\dot{\mu}_3+\rho\mu_3;\ \frac{\partial \mathcal{H}}{\partial h}=-\dot{\mu}_4+\rho\mu_4$$
$$\frac{\partial \mathcal{H}}{\partial \lambda_1}=-\dot{\mu}_1+\rho\mu_1;\ \frac{\partial \mathcal{H}}{\partial \lambda_2}=-\dot{\mu}_2+\rho\mu_2$$
$$\frac{\partial \mathcal{H}}{\partial \eta_1}=0;\ \frac{\partial \mathcal{H}}{\partial \eta_2}=0;\ \frac{\partial \mathcal{H}}{\partial \eta_3}=0;\ \frac{\partial \mathcal{H}}{\partial \eta_4}=0;\ \frac{\partial \mathcal{H}}{\partial \eta_5}=0$$
$$\frac{\partial \mathcal{H}}{\partial \mu_1}=\dot{\lambda}_1;\ \frac{\partial \mathcal{H}}{\partial \mu_2}=\dot{\lambda}_2;\ \frac{\partial \mathcal{H}}{\partial \mu_3}=\dot{k};\ \frac{\partial \mathcal{H}}{\partial \mu_4}=\dot{h},$$

plus eight boundary conditions: two for the initial values of h and k, two for the initial costates μ_1 and μ_2, and four transversality conditions for states (λ_1 and λ_2) and costates (μ_3 and μ_4) of this problem. Notice that since λ_1 and λ_2 are initially unconstrained, $\mu_{1,0}$ and $\mu_{2,0}$ are zero.

If we expand the above equations, we can immediately notice that it is possible to divide all of them by a suitable power of, say, y, in such a way that the newly normalized variables are constant in the steady state. Therefore, the steady-state allocation does not depend on the transition path. The optimal solution may then be found by obtaining the steady-state growth rates of all variables (as a function of the output growth rate γ) and substituting all time derivatives accordingly. It then suffices to divide all equations by a convenient power of y, and solve for all variables[4] the resulting nonlinear system of equations. For the sake of notation, assume that variable x is the ratio of x_t to some suitable power of output y_t in the steady state, so that x is a constant. Let us assume that the steady-state allocation P is an interior one, and also that $\eta_1=\eta_2=\eta_3=\mu_1=\mu_2=0$, $\eta_4=\eta_5$, and $\tau_k=\tau_w=0$. Using these assumptions and noting that F_k and F_h are homogeneous of degree 0, it is a tiring but straightforward task to show that all the above equations are verified if P also satisfies

$$U_c = U_{g_c}$$
$$G_{g_k} = G_i$$
$$H_{g_h} = H_{i_h}.$$

Equations (13)–(21) are still valid and determine seven components of P as a function of the remaining five. Since two instruments are posited (τ_k and τ_w), only three variables remain to be found. The previous three equations carry out that task. Because of the particular shape of functions U, G and H, the solution is an interior one. ■

A.2 Proposition 2

Proof. At time $t=0$ output is fixed. Therefore, capital and labor income taxation are lump-sum. If the government initially collects $\tau_{c,0}c_0$ in consumption taxes and $\tau_{k,0}=\tau_{w,0}=0$, then one point less in the consumption tax rate, compensated by a lump-sum tax of equal amount by increases in $\tau_{k,0}$ and $\tau_{w,0}$, impacts positively on welfare. Therefore, initially the government taxes capital and labor income as much as possible, perhaps up to the point where all efficiency conditions for public expenditure are met. Since the households respond to this by reducing investment, the government then substitutes income taxation, which must be zero in the steady state, for consumption taxation. ■

[4] Plus the steady-state output growth rate γ, since we have one equation left, $y=F(k,h)$, because of the normalization just made.

A.3 Proposition 3

Proof. We sketch the proof as follows. If $H_{i_h}(h, i_h, g_h) = 0$, then equation (18) reduces to $\rho + \gamma\sigma + \chi = \tilde{H}_h(h, g_h)$ with $\tilde{H}(h, g_h) = H(h, i_h, g_h)$. This equation defines h as a function of g_h alone, which means that τ_w does not distort the economy. Equation (17), $\rho + \gamma\sigma + \delta = (1 - \tau_k)F_k(k, h)\, G_i(i, g_k)$, implies that τ_k distorts the economy. Finally, the steady-state budget constraint, $g = (\tau_w(1 - \alpha) + \alpha\tau_k)\, F(k, h) + \tau_c c$, states that capital taxation can be painlessly exchanged for labor and consumption taxation, something which diminishes distortions. This is carried out to its maximum possible extent, that is, to the point where $\tau_k = 0$ and provided that the tax base is sufficient to cover the total public spending. ∎

A.4 Proposition ??

Proof. Notice that equations (16), (17) and (18) are linear in τ_k, τ_w and τ_c. Therefore, they can be solved in order to those variables as long as c, $G_i F_k$ and $H_{i_h} F_h$ are different from zero, which is always true. The solutions are

$$\begin{aligned}
\tau_k &= 1 - \frac{\sigma\gamma + \rho + \delta}{G_i F_k} \\
\tau_w &= 1 - \frac{\sigma\gamma + \rho + \chi - H_h}{H_{i_h} F_h} \\
\tau_c &= c^{-1}\left(g - \tau_k k F_k - \tau_w h F_h\right).
\end{aligned}$$

The remaining equations do not depend on any of the tax rates, and the maximization problem is thus independent of tax rates. ∎

A.5 Proposition ??

Proof. We see that in system (13)–(21) the consumption tax rate appears only once. By forming a Lagrangian function for the SSWM problem, a simple application of the envelope theorem shows that the total derivative of welfare (which is equal to the Lagrangian at the optimum) with respect to τ_c is equal to its partial derivative with respect to τ_c. This is easily seen to equal qc, where q is the government's constraint Lagrange multiplier, and is positive if additional costless resources for the government to spend are welfare-improving, that is, if q is positive. This in turn is always true as long as the unconstrained optimal g has not been attained. ∎