Optimal Asset and Debt Portofolios: an Hedging Strategy*

Abel M. Mateus† Sweder van Wijnbergen‡

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Abstract

There have been few contributions to the theory of the optimal portfolio of a central bank and national treasuries. In fact, most of the theory of optimal debt regards only the level of such debt and the theory of optimal portfolios although giving information on the currency composition does not apply to the national problem. This paper develops an integrated approach and gives special emphasis to the hedging strategies of monetary authorities to decrease the risk of external trade fluctuations. The model is then applied to the case of Portugal and simulations show that a significant improvement in the net external position of the monetary authorities can be achieved.

1 Introduction

The degree of uncertainty in international markets has increased substantially over the past few decades, essentially since the commodity market boom in the early seventies and the breakdown of the global fixed exchange rate system. As a consequence, countries have faced an increasing exposure to terms of trade risk. Figure 1 shows the terms of trade of Portugal over the period 1958-1995 which vividly demonstrates this. From 1972 to 1984 there was a terms of trade loss of about 40% followed by a terms of trade gain of about 20% between 1984 and 1987.

Should there be a concern about such volatility? Can the country isolate herself by managing the asset and debt position of the Monetary Authorities? There are two reasons to expect an affirmative answer. First, countries with less well developed capital markets and with less than perfect integration in world capital markets may not have ways to cushion themselves against this risk. In that case, transferring such risks to international market participants better set up to manage risk or more willing to take it can lead to obvious welfare gains.

The second point is more macroeconomic and has to do with the stability of the financial system. Highly variable reserve flows may lead to doubts about

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†Banco de Portugal and New Lisbon University

‡University of Amsterdam and London School of Economics

the sustainability of the exchange rate regime; if there will be a crisis, it is more likely to happen at times of low inflows than at times of high inflows; and such anticipations, even if there is no objective reason to expect a crisis, may in fact bring one about. Thus, smoothing foreign exchange exposure may well decrease the perceived and therefore actual risk of exchange rate crisis, with obvious benefits in terms of lower local interest rates.

To what extent does improved private access to international capital markets lowers the need to lay off risk into international capital markets, at the macroeconomic level? It is clear that large private international asset portfolios will have an impact on any national export volatility hedging strategy [Svensson (1990) for a formal example]. However, it is not clear that it obviates the need for such a strategy to begin with. It is true that if the whole exports volatility is hedged at the private level, then there is no need for a national hedging strategy[1]. There are many reasons, however, for doubting that such a completely privately instigated hedging strategy would ever come about. There are substantial costs to small exporters in devising and implementing such strategies, making it highly unlikely that they will be implemented. Moreover, the absence of long maturity derivative contracts would require individual hedgers to issue long term debt, something that is obviously impossible for all but a very few firms. Notice that the need for a hedging strategy has nothing to do with unexplored arbitrage opportunities; the objective of hedging is not to make money on average but to reduce risk, to reallocate risk to those better equipped or more willing to bear it. And the second argument, about how reduced volatility may reduce the risk of speculative attacks, stands if anything with stronger force is an open capital market, since speculative attacks are easier to launch in such an environment.

It is generally advantageous to lay off terms of trade fluctuations in international capital markets, but is it possible? One approach would be to actually minimize exposure by changing the structure of trade. Typically, most advanced countries have highly diversified trading patterns, limiting their exposure to much of the variation in cross-currency exchange rates and relative price movements. But restructuring trade patterns is obviously not possible in an essentially open trading environment dominated by private sector participants, and small open economies have usually a lower degree of diversification.

If exposure through trading patterns cannot be avoided, capital markets can be useful in shifting some of the resulting risk to other market participants. One approach would be to use derivative instruments such as futures contracts to hedge the risks mentioned. This is clearly possible in theory, but several problems arise. First there is the general problem of whether the instruments available in the market are sufficiently correlated with the particular variable (say export earnings) the country may want to hedge. In the end this is an empirical question, which we will investigate in this paper.

A second issue is the availability of such instruments to begin with. While there is an extensive set of derivatives available in international markets, their maturity usually does not stretch out beyond at most a year or maybe 18 months. Longer contracts would have to be negotiated in an exceedingly thin

[1] This is because the hedging strategy derived below is linear. Thus the sum of individual hedging portfolios actaully reproduces the aggregate portfolio. This would not be true for non-linear strategies or instruments, such as strategies involving options; the average of a set of options is not equivalent to an option on an average.

market and might thus be very expensive. Of course one can always hedge on the basis of short term contracts and simply roll them over, but this would limit the hedging strategy to the shifting of short run, high frequency fluctuations, as contracts cannot be rolled over at unchanged prices.

The final possibility open to any country with significant foreign debt and/or reserves is active currency management of its net reserves/debt position. This is a real possibility in Portugal and for other developed and developing countries. We regard this area as clearly underdeveloped theoretically. In practice, most of the management techniques for portfolios of central banks and Treasuries are based on purely commercial (or Markovitz) bank techniques, which is clearly unsatisfactorily. The objective pursued in our paper provides a framework within which to judge the question of what currency to issue foreign debt in, an increasingly important question as Portugal increases its presence as a borrower in international markets.

Our approach does not consider of importance short term hedging strategies. A short term hedging strategy almost certainly leads to continuously shifting hedging portfolios as new information comes in [See Claessens (1992) for an example]. Such strategy would be prohibitively expensive to implement because of the high transaction costs involved in changing the currency composition of existing debt on a continuous basis. While currency swaps offer the possibility of changing currency composition without actually renegotiating the terms of the existing debt, such a strategy is likely to be difficult, and certainly expensive to implement, particularly if there is a shortage of counterparts with roughly comparable credit risk rating.

Central banks (and Treasuries) should be concerned with ironing out longer term peaks and troughs rather than small day-to-day fluctuations; in statistical terms, this suggests one should be concerned with unconditional rather than conditional variance. Thus, the fact that the portfolio shifts required by a short term hedging strategy are too expensive when currency composition is used, may in fact not be a problem at all, since such a strategy is undesirable to begin with.

2 Optimal international portfolios

According to Stultz (1981) and Claessens (1988), the optimal portfolio for an international investor that can hold domestic or foreign bonds, and is subject to risk in exchange rates and the streams of payment of each bond, and that maximizes intertemporal consumption is given by

$$bW = \underbrace{\left[\frac{1}{1-\gamma}\right] V_{ee}^{-1}\nu\,[W]}_{A} - \underbrace{V_{ee}^{-1}V_{ef}\,[ef]}_{B} + \underbrace{\left(1-\frac{1}{1-\gamma}\right) V_{ee}^{-1}V_{es}\begin{pmatrix}\alpha\\0\end{pmatrix}[W]}_{C} \quad (1)$$

where

$$
\begin{aligned}
W &= \text{wealth}\\
\gamma &= \text{coefficient of risk aversion}\\
V_{ee} &= \text{variance-covariance matrix of exchange rates (NxN)}\\
V_{es} &= \text{variance-covariance matrix of exchange rates}\\
&\quad \text{and exogenous prices (Nx(N+K))}\\
V_{ef} &= \text{variance-covariance matrix of the changes in}\\
&\quad \text{exchange rates and the changes in the current}\\
&\quad \text{domestic currency values of the future receipts}\\
&\quad \text{of foreign currencies}\\
b &= \text{shares of wealth invested in foreign bonds (Nx1)}\\
\nu &= \text{vector of excess returns on foreign bonds}\\
ef &= \text{vector of domestic currency equivalents of the}\\
&\quad \text{current foreign currency values of the future payments}\\
\alpha &= \text{consumption shares (Kx1)}\\
0 &= \text{vector of zeros (Nx1)}
\end{aligned}
$$

This can be interpreted the following way: the first portfolio, A, is **the mean-variance or speculative portfolio**. Its composition depends on the expected return excess on the bonds and the inverse of the variance-covariance matrix of exchange rates. It is the traditional CAPM portfolio, which corresponds to the point of tangency of the capital market line and the efficient frontier of portfolios of risky assets in the space of nominal expected returns and the standard deviations of those returns for domestic investors. The higher the risk aversion coefficient,$1-\gamma$, the lower the investment will be in this mutual fund.

The **minimum-variance portfolio hedge,** B, is independent of the level of risk-aversion. The investor will borrow in foreign bonds as they can insulate him against changes in the domestic currency value of the future payments, caused by exchange rate movements.

And third, the **price-level hedging portfolio**, C, to insulate him against the risk that exchange rate covary with commodity prices.

Therefore, the optimal portfolio is a linear combination of a speculative and hedging portfolios, with higher weight for the last component in case there is a higher risk aversion[2].

In the following section we will estimate only the hedging portfolio.

Hedging reduces variance. However, a distinction must be drawn between conditional variance, which is potentially reduced by short-term hedging, and conditional variance, which is minimized by long-term hedging[3]. From a mone-

[2]Note that Central Banks, and Treasuries for the matter, have highly risk aversion guidelines for the management of her reserves. This means that a minimum-variance portfolio model is not liquely to be the most important instrument for these authorities. Besides, in practice, minimum-variance models are usually marred by problems of the choice of numeraire: the results are extremely sensitive to the choice of the numeraire, which by definition is an arbitrary variable.

[3]The former reduces the amplitude of month-to-month variations, while the latter irons out the major peaks and troughs.

tary authority vantage point of view, reduction in unconditional variance is the overriding objective.

3 Estimation

In this section we will estimate a long run hedge of Portugal's deflated export revenues, following Dumas and Jorion (1993). Available instruments in constructing the hedge rely on German mark, French franc, British pound, US dollar, and Japanese yen exchange rates as well as gold prices. Consequently we will search for an optimal hedging portfolio, in terms of a net position in reserves/external debt for each currency and gold. We first estimate a long run relationship, by cointegration, between log deflated export values and log deflated exchange rates and the log gold price. The results provide an optimal hedge. Next, we investigate implementation of the theoretical nominal and replicator hedge by means of simulations in order to determine potential gains.

The structural equation underlying the long run hedge is given by the cointegration relation:

$$\log y_t = \log T_t + \sum_i \beta_i \log x_{i,t} + \epsilon_t \tag{2}$$

where y_t are deflated exports, $\log T_t$ represents the regression constant, trend, and seasonal terms, and $x_{i,t}$ are deflated hedging instruments. Subscript t denotes time, and i labels instruments. We are interested in hedging long run fluctuations in the detrended and deseasonalised deflated export value.

By construction, the theoretically correct hedge is given by

$$\prod_i x_{i,t}^{\beta_i} \tag{3}$$

The composition of the optimal portfolio is given by the coefficients β_i, which are proportions in terms of total exports. Without taking a view as to where exchange rates or gold prices are going, this relationship allows us to construct a financial hedge, such that an increase in the value of external debt offsets an increase in the value of exports, and the converse is also true. The β_i coefficients may be interpreted as exposures (or sensitivities) of the y variables to the movements in the x variables, or, in this case, foreign-exchange exposures[4]. For example, a positive coefficient on the value of the US dollar implies that the dollar should be sold forward or that some external debt should be denominated in dollars. Notice that these coefficients are net positions, so they refer to external debt (liabilities) minus reserve (asset) positions for the monetary authorities (Central Bank plus Treasury). Thus, a negative coefficient means that there should be a net creditor position in that particular currency or item. If, for a particular currency the net exposure coefficient is null it means that the monetary authorities should balance its position: if there is an external debt in that currency, there should be an equivalent asset position in that particular currency.

[4] Contrary to Dumas and Jorion, Claessens and others we do not include as hedging instruments interest rates. We believe that in the long-run interest rate movements will be strongly correlated with exchange rate movements (interest rate parity holds) so we do not need to include those in the right-hand variables.

Estimation is carried out in four steps:

1. We first establish the I(1) nature of the time series by means of Phillips-Perron (PP) tests.

2. Then, we iterate with various sets of instruments: (i) investigation of cointegration by means of Dickey-Fuller (DF) test, and (ii) estimation of Phillips-Loretan (PL) equation (with autocorrelation correction) with GARCH correction.

3. Next, the performance of the (theoretical) long run hedge can be assessed by substitution of the relevant parameter values, estimated by the Phillips-Loretan regression, in the structural, or cointegration, equation.

4. Finally, we estimate the equation in first differences, which is supposed to give a short term hedge, minimizing the conditional instead of unconditional variance.

The Phillips and Loretan (1991) procedure is required because there may exist feedback correlations between exports and exchange rates, reflecting, for instance, balance of trade disequilibrium. In this case the right-hand variables are not properly exogenous. The Phillips-Loretan method corrects for this by adding leads and lags of first differences of the regressors which asymptotically accounts for endogenous regressors. In addition, to correct for serial correlation in the residuals, a lagged value of the equilibrium error is added to the equation. This equation has to be estimated by non-linear least squares.

The Portuguese export, exchange rate, gold price, and deflator data cover 72 quarters from 1978.I-1995.IV. Exchange rates are the fixing rates by the Banco de Portugal, price of gold is the price in the London market and the export and import data is from Direcção Geral do Comércio Externo of the Ministério da Industria e Comercio. Figure 2 plots the deflated data series in both logs and first differences of logs. We chose to deflate all variables by the price index for imports. This choice of a deflator is rooted on the idea that, were Portugal to have one more unit of income to spend, it would want to spend it by importing some more of the same basket of goods it is currently importing.

All variables seem to be I(1) according to PP tests, but evidence for cointegration from the DF test is not too strong (acceptance of nonstationarity at 5% level). Results from the DW statistic are slightly better.

There seems to be significant multi-colinearity among exchange rates. That is why there is no apparent improvement in increasing the number of hedging instruments beyond a few. The best results are observed for the Dutch mark, US dollar and gold and for Dutch, Japanese yen and gold. Estimates for more instruments are generally insignificant. Other European currencies do not seem to have any role in the hedging portfolio. The results of the cointegration relation with all instruments and a trend (besides the seasonals and a constant) are:

Instruments	Coefficients	T-Statistics
Trend	0.02124	8.0228
LDEM	1.6411	6.9908
LFRF	-0.9185	-5.0038
LGBP	0.2851	1.3481
LUSD	-0.3444	-2.3665
LJPY	-0.2923	-2.5491
LGOLD	-0.1184	-2.1954

$$R^2 = .9877, N = 72, DW = .8486, Q = 88.29$$

With the T-Stat for residuals -3.548, without significant further differenting of residuals[5].

The results of the Phillips-Loretan equation with one lag and one lead are:

Instruments	Coefficient	T-Statistic
DEM	0.8707	2.3624
FRF	-0.7464	-3.3627
GBP	0.4272	1.4157
USD	-0.6223	-3.4072
JPY	0.2312	1.2125
GOLD	-0.0830	-0.9733

$R^2 = .996, Q(4) = 3.2931$, corrected for autocorrelation.

As we can see the coefficients are not all significant. In the optimal portfolio DEM, GBP and JPY should be bought forward and the other currencies, including gold sold forward. Notice that gold has almost a null coefficient in this equation.

The following are estimation results for the cointegration and PL regressions respectively for our preferred equation:

Instruments	Coefficients	T-Satistics
DEM	0.5983	7.2921
USD	-0.5985	-4.3125
GOLD	-0.2519	-4.5598

$R^2 = .9819, DW = .834, DF_2 = -3.6458, N = 72$

And the Phillips-Loretan estimation with one lead and one lag:

Instruments	Coefficients	T-Statistics
DEM	0.5260	3.6256
USD	-0.6174	-2.732
GOLD	-0.2438	-2.3512

$$R^2 = .9928, N = 70, var(\log \hat{y}) = 0.000468, var(\textstyle\sum_i \beta_i \log x_i) = 0.000342,$$
$$var(\log \hat{y} - \textstyle\sum_i \beta_i \log x_i) = 0.000213$$

If we increase the number of leads and lags in the P-L regression the US dollar looses significance. Figure 2A plots the residuals from the cointegration regression[6]. Figure 3 shows autocorrelations in the residuals from the Phillips-Loretan regression. In a previous version of the paper with the sample up to 1992 the results showed that a portfolio with DEM and gold had the best performance. However, the cointegration relation with the larger sample were somewhat less satisfactory. This shows that including the last three years changes structurally the model and makes a portfolio with the US dollar and DEM a better performer. Gold seems to become also more important as an hedge instrument. Introducing the more recent years counterbalances significantly the period of the early eighties that had large fluctuations of gold prices.

[5] DEM means Dutch mark, FRF French frank, GBP British pound, USD US dollar, JPY Japanese yen and GOLD gold. These terms preceded by L mean logs of the time series.

[6] Figure 2B plots the residuals for a DEM-JPY-GOLD portfolio.

Mixing the DEM either with the GBP or the FRF does not perform well in the cointegration. Finally, we found another portfolio that does not perform badly, with the DEM, JPY and gold, were the Japanese yen takes the role of the US dollar. However, the results are still worse than our preferred portfolio.

Substituting the estimation results of the PL regressions in the structural hedging equation we can evaluate the performance of the log (theoretical) hedge. The log DM-USD-gold hedge is able to cover more than half of the variance in export earnings[7]. Thus we conclude that the performance of the theoretical hedge is acceptable.

What about short term or conditional variance hedging? Figure 4 gives percent changes in real export values and the instruments from the hedges. From our estimation it appears that about one third of the variance in export revenues can be explained by fluctuations in the DEM and USD exchange rate and gold prices. In conclusion, there seems to be more scope for short run hedging than in Dumas and Jorion (1993). Although the DEM-USD-gold portfolio performs slightly worse in the short run, it seems a more effective long run hedge.

Implementing the logarithmic hedge would require contracts with continuously compounded returns. These can however be approximated by one-period contracts that are rolled over and updated every period t. Then, the logarithmic hedge of deflated exports $\hat{y}_t$ is replaced by[8]

$$\hat{y}_0 + \sum_{\tau=0}^{t-1} \hat{y}_\tau \left[\sum_i \beta_i \frac{x_{i,\tau+1} - x_{i,\tau}}{x_{i,\tau}} \right] \qquad (4)$$

We should also recognize that real contracts like this cannot be implemented, since only nominal instruments are available. Instead, consider an hedge in which (real) returns on real instruments are replaced by deflated returns on nominal instruments:

$$\hat{y}_0 + \sum_{\tau=0}^{t-1} \hat{y}_\tau \sum_i \beta_i \left[\frac{X_{i,\tau+1} - X_{i,\tau}}{X_{i,\tau}} - \frac{P_{\tau+1} - P_v}{P\tau} \right], \qquad (5)$$

where P_t is the appropriate deflator, and $X_{i,t} \equiv P_t x_{i,t}$. Clearly, this is only a minor departure from the real hedge, stemming from a dynamic approximation.

Finally, we performed several simulations within the sample and out-of-sample based on the chosen hedging portfolio. First, we evaluate replication of the theoretical hedge by means of a hedge by one-month contracts which are rolled over every month. Second, we try a replicating hedge using nominal instead of deflated instruments. Using the DEM-USD-gold portfolio Figure 5 shows that the theoretical log hedge can be replicated reasonably well by one month rolling-over hedges. In Figure 6 we conclude that both hedging portfolios track $\hat{y}_t$ well in the second half of the period. This is confirmed by a plot of residual gains ($\hat{y}_t - hedge$; Figure 7). Similar conclusions can be drawn from

[7]Note that this variance decomposition is not complete. The parameters from the PL regression are not OLS parameters of the structural equation, so the covariance between the log hedge and the residual is likely to be nonzero.

[8]The approximation argument is the following. We know from (1) that, apart from errors, and for time τ, $\Delta \log \hat{y}_{\tau+1} = \sum_i \beta_i \Delta \log x_{i,\tau+1}$. Thus, $\hat{y}_{\tau+1} = \hat{y}_\tau \exp(\sum_i \beta_i \Delta \log x_{i,\tau+1})$ which can be approximated by $\hat{y}_\tau(1+\sum_i \beta_i \Delta x_{i,\tau+1}/x_{i,\tau})$. Iterating for $\tau = 0, ..t-1$ produces (3).

out-of-sample simulations (Figures 8-10). In these simulations, parameters re-estimates from a subsample consisting of the first 64 quarters are employed to simulate the hedges both in sample for the first 64 quarters and out-of-sample for the last two years. Hedges based on deflated pay-offs of nominal instruments (Figures 11-16) have slightly more problems with tracking the log hedge, but the performance is still acceptable.

4 Conclusions

We have studied and estimated an optimal portfolio for hedging external shocks to the economy. This has been a largely neglected area in the determination of an optimal international portfolio for monetary authorities. Traditionally, most of the attention has been directed towards the speculative portfolio, or Markovian type of portfolio. In fact, for monetary authorities that should have a high risk aversion and be concerned with macroeconomic objectives, the hedging portfolio seems to be of an overriding concern.

In this study we identify very few instruments as required for an hedging portfolio. In the Portuguese case a DEM-USD-gold portfolio would do the job. Simulations for in-sample and out-of-sample show good results and more than half of external shocks in export revenues can be ironed out through the hedging portfolio.

Portugal has been experiencing a substantial change in her export structure. Those changes in structure can be accommodated by estimating hedging portfolios restricted to groups of products. However, there may be a problem if there is no history, i.e., if there are new industries and new products.

It is important to emphasize that we have dealt with only one aspect of the management of asset/public debt. Among other important issues of major relevance are the incentive compatibility of asset/public debt management and inflationary expectations, as well as exchange rate stability. There are important policy implications of that analysis, as e.g. Calvo (1996) has shown, for the fixed versus indexed interest rates, short or long duration, and internal versus external debt.

Besides an optimal international portfolio, central banks are also concerned with reserves required for intervention in the foreign exchange market for exchange rate management, in our case, within a target zone regime. This problem can be handled by adding an exogenous amount to the restriction of our optimal problem.

Finally, let us refer the interesting problem of transition towards monetary union that raises important questions in terms of the currency composition of assets/external debt, as well as the post-euro compared with the pre-euro situation. However, those are issues for another paper.

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