Going public when market information is valuable for insiders.

Sérgio Lagoa (ISCTE)* and João Amaro de Matos (Univ. Nova de Lisboa)

October 2005
* Corresponding author: sergio.lagoa@iscte.pt
Abstract

Consider a firm that undergoes a project with the help of an active investor, after which it decides to go public. This paper studies the going public decision focusing on two main aspects. First, we study the impact of the exit strategies by the active investor on the decision to go public. Second, we study how managers’ uncertainty about the quality of the firm after the project, as perceived by outsiders, may impact the decision to go public. We assume that the management is composed by an entrepreneur and an active investor. The entrepreneur is not interested in exiting the firm and the active investor is a professional temporarily committed to the firm, willing to exit after the project is finished. We also assume that the management does not know for sure the quality of the firm after the project is completed. The management, who knew the firm’s type before the project, may be uncertain about the final quality after the project is completed. We find that a firm goes public only if the cost of its evaluation for outsiders is sufficiently low. Our model also implies three additional results. First, the firms increase their preference for the public market as the public market dimension increases. Second, as the active investors increase their control over the management, the firms tend to go public more often. Third, the model predicts that, depending on the parameters of the model, firms that decide to go public tend to out-perform the market average.
1 Introduction

A firm with no shares transacted in the public equity market is said to be private. When a private firm issues shares in a public market, we say that the firm is going public. The choice of going public, either for the first time or by reversing a LBO, may be explained in different contexts. The main reasons pointed in the literature are, among others, the financial needs to invest in new projects, the reduction of agency costs of debt (Myers, 1977; Galai and Masulis, 1976), the reduction of managerial ownership (Fama and Jensen, 1983, 1985; Demsetz, 1983; Morck, Shleifer and Vishny, 1988; Habib, 1997), the increase in portfolio diversification of the entrepreneur (Pagano, 1993; Shah and Thakor, 1988), and the reduction of excessive monitoring by large shareholders and banks (Rajan, 1992; Pagano and Roëll, 1998). Additionally, we also consider that the investors in the private firm may feel that their effort of organization and consolidation of the strategies and processes that improved the competitive position of the firm is complete. At that point, they may wish to collect a return on their investment (Mian and Rosenfeld, 1993; Pagano, Panetta and Zingales, 1998).

In this paper we study the going public decision, focusing on two main aspects. First, we study the impact of the exit strategies by significant insiders on the decision to go public. Second, we study the impact of the management’s uncertainty about the quality of the firm when deciding whether or not to go public.

Regarding the first issue, Mian and Rosenfeld (1993) highlight the importance of the exit strategies by active investors in Reverse LBOs. They found that many of the firms that reverse LBOs are taken over with a good performance after going public. These authors provide empirical evidence that this takeover activity is the result of exit strategies by active investors. Therefore, one important motivation for firms to go public may be the fact that some insiders want to get out of the firm. Pagano, Panetta and Zingales (1998) arrive to similar conclusions for the case of Initial Public Offers.

We now focus on the second issue. In any private firm the management has a subjective view of the firm’s value and thus, cannot be truly sure about how the market will react to a public offer. This is true both for the case of an IPO as for a reverse LBO. In particular, after a LBO, a firm undergoes a reorganization process that may lead to a change in its quality. The insiders could be perfectly aware of the firm’s quality before the reorganization, but there will be some uncertainty about the market reaction once the reorganization is completed. When such firms have new projects and seek financing, markets end up providing additional information, reducing this uncertainty. When going public, firms signal their quality to the
market by their share prices. One of the possible explanations for long run underpricing of public offers is the pooling behavior from bad firms which, in turn, may be induced by wrong beliefs about their true quality.

In this paper, we attempt to model the determinants of the going public decision. Former models that also stress the importance of the information structure in the going public decision are, among others, those by Chemmanur and Fulghieri (1999), Subrahmanyam and Titman (1999) and Maug (2001). Our model, however integrates the empirically relevant issue of the existence of active investors and the uncertainty of insiders about the firm’s quality. This allows to characterize many conclusions out of the reach of the above cited models.

Among other well known results, our model shows that a firm goes public only if the cost of its evaluation for outsiders is sufficiently low. More importantly, our model implies four unique results due to our setting.

First, as the public market dimension increases, the critical value of the evaluation costs below which firms go public, will increase. This result explains why the average age of European companies that go public is higher than the average age of the US companies that go public, since the US stock market dimension is much larger than the European stock market.

Second, as the active investors increase their control over the management, the critical value for the costs of evaluation below which firms go public, will also increase. This is consistent with the empirical evidence of Jelic, Saadouni and Wright (2001), that venture backed MBOs tend to go public earlier than non-venture backed MBOs.

Third, the model predicts that firms may on average out-perform the market after going public. This result explains the empirical evidence of DeGeorge and Zeckhauser (1993), Holthausen and Larker (1996) and Mian and Rosenfeld (1993), that reverse LBOs present, after they go public again, a long-run performance above the return of their comparison firms.

Fourth, as firms increase their investment risk and/or concentrate their focus on their core business, the critical value for the evaluation costs, below which these firms go public, will increase.

The rest of the paper is organized as follow. Section 2 describes how we model the firm and the agents, namely what is the decision time line, the initial information of insiders, the investment technology, the evaluation technology of outsiders and the characteristics of the VC and agents in the public market. Section 3 describes the private equity market and the VC financing conditions. Section 4 describes the public market and the conditions under which the firms go public. Section 5 models the information

---

1See e.g. Chemmanur and Fulghieri (1999).
process, characterizing how the insiders collect information from the financing process, allowing to compare the equilibria in the two former markets. Finally, Section 6 compares both equilibria and explains the choice of the management between going public and the VC financing. All proofs are in the Appendix 8.

2 The Firm and the Agents

We assume that a firm is a set of ongoing projects, that have been financed in the past. The management is composed by an entrepreneur, who is not interested in exiting the firm, and an active, professional investor, who helped financing one specific project (to be referred to as the active project) and is willing to exit once this project attains a steady-state.

Consider that one such private firm has access to a new project (to be referred to as the new project), and needs external financing. The new project may be financed either by going public or by selling shares to a Venture Capitalist (VC). We assume that there are two different types of firms: good firms and bad firms. The true type of the private firm is not know for sure neither by insiders nor by outsiders before the financing of the new project. Outsiders may evaluate the firm at a given cost and transfer this information to insiders during the financing process.

In the private equity market we assume that the VC is risk-averse, and will not accept to finance the project without evaluating the firm. Therefore, the firm cannot offer a contract without information to the VC.

In the public market some investors do produce information whereas others may try to free-ride and do not produce any information. This problem is typically solved in the markets by the intermediation of investment banks. In our context, the role of investment banks would be to provide information about the firm to investors, lowering the costs of going public. However, investment banks do not completely eliminate the information asymmetry between insiders and outsiders for two reasons. First, the information provided by the bank may be incorrect and must still be checked by the investors. Second, the bank may disclose only part of its information about the firm, giving its interest in controlling the public offer. Hence, even the presence of investment banks does not avoid the need of some external evaluation and related costs. The practical impact of investment banks in our model would simply be a lower value of evaluation costs for outsiders. Thus, for modelling simplicity, we assume that the role of investment banks may be ignored and is replaced by the assumption that the agents have always advantage in pro-
ducing information.\footnote{In the conclusion we discuss the existence of investment banks may affect our main results.}

Finally, and for simplicity, we assume that bad firms have always a pooling strategy with good firms, meaning that both type of firms offer the same number of shares at the same price\footnote{This assumption avoids the additional difficulties arising from an unnecessary comparison with a separating equilibrium. Our results seem to be robust with respect to that comparison.}. In the next sections we describe the timeline of the model, describe what insiders know about the firm’s type, characterize the technologies of investment, and model the information technology. Next, we model the behaviour of the VC and outside investors in the public offers.

\subsection{The model timing}

The model has four periods of time. At the initial time \( t = 1 \), after the active project has attained the steady state, the firm has access to the new project and needs external financing. The firm decides to undergo the project choosing to be financed either by the VC or by the public market, offering in either case specific conditions. In the case of the VC, the management offers a fraction of equity to be sold, conditioned on the results of a future evaluation of the firm to be done by the VC. In the case of the public market the contract conditions are basically the definition of the share price and number of shares to be sold.

At time \( t = 2 \), outsiders decide about financing or not the new project. If the firm has decided to go public, the investors in the public market choose at this point to invest or not in the firm, allowing the firm to know the outcome of the public offer, namely the amount of funds raised. Alternatively, if the VC financing has been chosen by the firm, then the VC decides at this point whether or not to finance the firm. If the VC’s answer is positive, then he or she proceeds to the firm’s evaluation and transmits the obtained result to the firm.

At time \( t = 3 \), after observing the outcome of the financing process, the active investor evaluates the decision of leaving the firm and either leaves or does not leave.

At time \( t = 4 \), the cash flows associated to the whole set of projects (old and new) are known. Notice that such cash flows depend on the type of the firm.
2.2 The initial information of insiders

We first describe what insiders know about the firm’s type before and after the active project. During the private period the firm goes through important changes while the active project is implemented, possibly with changes in the management and with reorganization and/or consolidation of strategies and processes. This implies the existence of two distinct states in the firm’s life: before the active project starts (\textit{ex-ante} state) and after the active project attains a steady-state (\textit{ex-post} state).

We assume that there are two different types of firms: good and bad. The proportion of \textit{ex-ante} good firms in the market is $\phi$. Among these, we denote by $\phi_1$ the proportion of those that remain good firms \textit{ex-post}. Analogously, a proportion $\phi_2$ of the \textit{ex-ante} bad firms become good firms. Furthermore, we assume that $\phi_1 > \phi_2$. All participants, both insiders and outsiders, are aware of the generic structure above. In particular, they know the values of $\phi, \phi_1$ and $\phi_2$.

Outsiders do not know the type of any firm without incurring in a cost of information. The entrepreneur and the active investor however, know their firm’s \textit{ex-ante} type. More precisely, given the available information at that point in time, they know exactly the potential quality of the active project. However, they do not know the firm’s \textit{ex-post} type with certainty, once the active project attains a steady-state. This occurs because the firm is private and they receive only a partial feedback from the market about the result of the active project undertaken. In the case of a LBO, where the active project may correspond to the restructuring of the firm, this effect is more clear. In fact, before the LBO, the firm would be receiving systematic information from the market. Once the firm becomes private, such flow of information ends and, after a certain point, the type evaluation becomes seriously uncertain.

2.3 Investment technology

Since at $t = 4$ the old projects are in a steady-state\textsuperscript{4}, we assume that they continue generating cash flow at that time, but do not need any additional investment.

The cash flow generated by any project in the firm depends on several factors: the amount invested, the quality of the firm and an uncertainty component. In general, for each project, the cash flow is proportional to the investment, plus a random term with zero mean. We take $k_q$ as the proportionality constant, reflecting the firm’s type $q$, where $q = G$ for good firms and $q = B$ for bad firms. Good firms have larger expected cash flow.

\textsuperscript{4}Notice that the old projects are in steady-state since $t = 1$. 

5
than bad firms ($k_G > k_B$). Let $D$ denote the investment done in the past on ongoing projects and let $t$ denote the amount to be invested in the new project. We assume that there is a critical value $I$ for the amount invested in the new project, above which no additional value is created. Thus, no one will invest an amount larger than $I$ in the new project. Furthermore, let $\hat{e}_D$ and $\hat{e}_I$ denote the mean-zero random variables modeling the uncertainty in the old and new investments respectively. Both the realization of $\hat{e}_D$ and $\hat{e}_I$ are unknown to insiders and outsiders. The time $t = 4$ cash flow from the firm is thus given by the following investment technology, that is known by the outsiders:

$$ v_q(t) = k_qD + \hat{e}_D + \hat{e}_I + k_q \begin{cases} t & \text{for } t < I \\ I & \text{for } t \geq I \end{cases} $$

(1)

where $\hat{e}_D \sim (0, \sigma_{e_D}^2), \hat{e}_I \sim (0, \sigma_{e_I}^2), q \in \{G, B\}$, and $k_G > k_B > 1$.

The variance of the firm’s cash flow depend on the variances of the cash flows of the old and new projects ($\sigma_{e_D}^2$ and $\sigma_{e_I}^2$, respectively) and on the covariance between them, denoted by $\text{cov}(\hat{e}_D, \hat{e}_I) \equiv \sigma_{DI}$.

We assume that the net present value of a firm’s new project is sufficiently large (i.e., $k_G$ and $k_B$ are large enough), so that the entrepreneur will always be willing to invest $I$. At the full investment level, the expectation at $t = 4$ of the future cash flow is

$$ V_G = k_G(D + I) $$

(2)

for the good type and

$$ V_B = k_B(D + I) $$

(3)

for the bad type.

### 2.4 The evaluation technology of outsiders

In our model, outsiders can evaluate the firm’s quality at a fixed cost $c$. Let $e$ denote the result of one such evaluation. We assume that only two outcomes are possible, either $e = g$ (the firm is evaluated as good) or $e = b$ (the firm is evaluated as bad). The evaluation is not necessarily accurate. We model the precision of the evaluation technology as follows

$$ \Pr(e = g \mid q = G) = 1 \text{ and } \Pr(e = g \mid q = B) = y, \text{ where } 0 < y < 1. $$

(4)

The evaluation technology is assumed to be the same for all outsiders. The outsiders’ evaluation cost depends on two important factors. First, it depends on the amount of information available in the market place about the firm and its management. The evaluation cost should therefore decrease as
the firm is older and more information about the firm is available. Second, the cost may depend on the industry. Firms belonging to industries intrinsically more difficult to evaluate, have a larger evaluation cost. The first factor is related to information availability, whereas the second is related to the difficulty in processing the information.

2.5 The venture capitalist and the investors in the public offers

We assume without loss of generality that there is only one risk-averse VC that can finance the entire new project. The VC is assumed to invest a large proportion of his/her wealth in the firm, meaning that the VC is not sufficiently diversified. Hence, the VC will demand a underdiversification premium from the investment in the firm.\textsuperscript{5} We assume that the capitalist invests his/her wealth only in the project and in the risk-free asset. Because the VC is assumed to be risk-averse, his/her objective is to maximize the utility from the cash flow received at time $t = 4$. The utility function is assumed to be of the form:

$$U(W) = \mu_w - \rho \sigma_w^2,$$

where $\mu_w$ is the mean and $\sigma_w^2$ is the variance of the wealth $(W)$ at time $t = 4$, and $\rho$ the coefficient of risk-aversion.

Consider the case where the management proposes a financing contract to the VC. The terms of this contract may be rejected or accepted. The VC will accept the contract if and only if the management’s offer leaves him/her at least as well off, in terms of utility, as investing in the risk-free asset.

Let us now consider the case where the firm goes public in order to finance the new project. It is assumed that each outside investor potentially financing the new project will buy either zero or one share of the firm in the public offer market\textsuperscript{6}. This means that each investor holds only a small fraction of the firm’s equity. Thus, there is no loss of generality assuming that the investors are risk-neutral\textsuperscript{7}. Since the investors are assumed to be risk neutral, their

\textsuperscript{5} An alternative modelization that produces the same results is to assume that the VC has bargaining power relative to the entrepreneur, enabling him to extract a fraction of the net present value of the firm’s project. The numerous investors in the public market have less bargaining power than the VC. This is analogous to a point raised by Chemmanur and Fulghieri (1999).

\textsuperscript{6} We assume that no investor is wealth constrained.

\textsuperscript{7} Our results will be qualitatively unchanged if the investor were also risk-averse. The crucial difference between the VC and the investor is that the latter has a greater share of his wealth tied to the firm, so either the VC demands a higher nondiversification premium.
objective is to maximize the expected value of their cash flow at time \( t = 4 \). As in the case of the VC, we assume that investors place the remaining wealth in the risk-free asset.

The decision about how to finance the new project may thus be seen as a game where the players are the management and the outsiders, namely the VC and the investors. In what follows we are going to study the Perfect Bayesian Equilibrium of this game.

3 Equilibrium in the private equity market

In this section we describe the private equity market and the VC financing conditions. If the private equity financing is chosen, the firm offers the VC a contract with information production. After that, the VC evaluates the firm. The management observes the outcome of the evaluation, and decides on the fraction of equity to offer the VC. Finally, the VC can only accept or reject the offered contract. We now verify under what conditions the firm offers the VC a contract with information production. The probability that a firm is \textit{ex-post} bad is

\[ \theta = \phi(1 - \phi_1) + (1 - \phi)(1 - \phi_2). \]

(6)

This is the probability that an uninformed outsider uses in his or her investment decisions. If an outsider conducts a costly evaluation of the firm, he or she will update \( \theta \), using the Bayes’ rule, as follows

\[ \Pr(q = G \mid e = g) = \frac{(1 - \theta)}{(1 - \theta) + y\theta} > 1 - \theta \quad \text{and} \quad \Pr(q = G \mid e = b) = 0. \]

The VC’s expectation of firm’s value at time \( t = 4 \), at full investment level \( I \), conditional on the outcome of the evaluation is

\[ V_e = E(k_q I \mid e), e \in (g, b). \]

and the information-based uncertainty is measured by \( \text{var}(k_q I \mid e) = \sigma_e^2 \), for \( e \in (g, b) \). When the firm is evaluated as bad, there is no information-based uncertainty, \( i.e., V_b = V_{B} \) and \( \sigma_b^2 = 0 \).

Let \( s_b^* \) denote the lowest share of the firm’s equity to the VC such that he or she accepts to finance the firm in case of a good evaluation, and \( s_b^* \) denote the equivalent minimum share in case of a bad evaluation. These fractions or has more bargaining power. This point was also stressed by Chemmanur and Fulghieri (1999).
of equity must leave the VC indifferent between investing in the firm and investing in the risk-free asset, in terms of expected time \( t = 4 \) utility. Thus, if the evaluation is good, \( s_g^* \) must satisfy

\[
s_g^* V_g - \rho s_g^{*2} (\sigma_g^2 + \sigma_{\varepsilon_D}^2 + \sigma_{\varepsilon_I}^2 + 2\sigma_{DI}) = I + c; \tag{7}
\]

and if the evaluation is bad, \( s_b^* \) must satisfy

\[
s_b^* V_b - \rho s_b^{*2} (\sigma_{\varepsilon_D}^2 + \sigma_{\varepsilon_I}^2 + 2\sigma_{DI}) = I + c. \tag{8}
\]

Our main result characterizing the equilibrium in the private equity market is as follows.

**Proposition 1 (Private Equity Market Equilibrium)** There is a critical value \( \rho_p \) such that, if the VC is not too risk averse (i.e. \( \rho < \rho_p \)), and if the active project of bad firms are relatively successful (\( \phi_2 > y/(1 + y) \)), then the equilibrium where firms choose private equity financing includes both types of firms offering the VC a financing contract with information production (with the equity fractions \( \{s_g^*, s_b^*\} \)), and the VC accepts the contract.

According to this result there exists a pooling equilibrium where both types of firms offer the same contract \( \{s_g^*, s_b^*\} \) to the VC, contingent on the evaluation to be performed by the VC. Underlying this equilibrium there are important assumptions about the circumstances under which firms may deviate from pooling. We assume that if a firm offers an information production contract with parameters other than \( \{s_g^*, s_b^*\} \), the VC infers that the firm is *ex-ante* bad with probability 1. Notice that the VC’s uncertainty about the *ex-post* type remains when firms deviate from equilibrium, since the insiders themselves do not know the firm’s *ex-post* type and, therefore, their actions cannot reveal the truth. Under these out-of-equilibrium conditions, the firm continues to offer a contract with information production, but with parameters other than those specified in the equilibrium contract.

If the evaluation is good, under the out-of-equilibrium conditions the firm offers the equity share \( s_{gB}^* \) satisfying

\[
s_{gB}^* V_{gB} - \rho s_{gB}^{*2} (\sigma_{gB}^2 + \sigma_{\varepsilon_D}^2 + \sigma_{\varepsilon_I}^2 + 2\sigma_{DI}) = I + c, \tag{9}
\]

where

\[
V_{gB} = E \left( k_q I \mid e = g; s_{gB}^* \right), \sigma_{gB}^2 = var \left( k_q I \mid e = g; s_{gB}^* \right)
\]

and

\[
Pr \left( q = G \mid e = g, s_{gB}^* \right) = \frac{\phi_2}{\phi_2 + y(1 - \phi_2)}.
\]
Under a bad evaluation, the firm offers the equity share $s_{bB}^*$ satisfying

$$s_{bB}^* V_{bB} - \rho s_{bB}^2 \left( \sigma_{zD}^2 + \sigma_{zI}^2 + 2\sigma_{DI} \right) = I + c$$

(10)

where

$$V_{bB} = E \left( k_q I \mid e = b, s_{bB}^* \right) = V_B = k_B I,$$

reflecting that under a bad evaluation, knowledge that the firm was \textit{ex-ante} bad does not increase the VC information. Notice that this out of equilibrium solution leads to $s_{bB}^* = s_b^*$ but $s_{bB}^* \neq s_b^*$. Actually, in Appendix A it is shown that $s_{bB}^* > s_b^*$, leading to a pooling equilibrium. In fact, good firms are penalized if they choose to separate because they are identified as \textit{ex-ante} bad firms; if they choose to pool, they will always obtain a good evaluation. Bad firms also do not have advantage in separating because they are identified as \textit{ex-ante} bad firms. However, if bad firms choose to pool, they may be evaluated as good with some positive probability, and sell their equity at a high price.

The value of the parameters of the model have an impact in the equity’s fraction to the VC. Such impact is described in the following corollary.

\textbf{Corollary 2 (Comparative Statics)} \textit{The fraction of equity that has to be offered to the VC in return for the financing required is (a) increasing in the cost of evaluating the firm, $c$; (b) increasing in the risk of old and new firm’s projects, $\sigma_{zD}^2$ and $\sigma_{zI}^2$; (c) increasing in the covariance between the cash flows of the old and new projects, $\sigma_{DI}$; (d) decreasing in the investment done by the firm in the past, $D$; (e) decreasing in the productivity of the firms, $k_q$; and (f) increasing in the VC’s coefficient of risk aversion, $\rho$. Finally, the impact of an increase of $I$ on the VC’s fraction of equity is ambiguous.}

Let us provide the intuition for such results. Regarding (a), if $c$ increases, the VC will have to support larger evaluation costs, and a corresponding larger fraction of equity will be required to finance the firm.

An increase in the risk of the old or the new projects would reduce the utility of the risk-averse VC. This would induce the VC to demand a higher compensation, leading to result (b).

For the same level of risk of the old and the new projects, an increase in the covariance between the cash flows of the old and new projects would increase the global risk of the firm, causing the VC to demand a higher equity to finance the firm. This is result (c).

With respect to (d), if the investment done in the past ($D$) were larger, then the cash flow of the firm at $t = 4$ would increase. This would have occurred without the need of any additional investment from outsiders at
$t = 2$, implying that the VC would demand a lower fraction of the equity, because he or she would already benefit from the higher investment done in the past.

Result (e) is explained as follows. If the firms were more productive, they would originate higher cash flows with the same investment. Therefore, because firms would have a higher value, the fraction demanded by the VC would be smaller.

Result (f) follows from the fact that an increase in the risk-aversion coefficient leads the VC to demand a higher compensation to support the same risk to finance the firm.

The ambiguity about the impact of an increase in the level of investments in new projects contrasts with the result in Chemanur and Fulghieri (1999), who conclude that an increase of $I$ decreases the VC’s fraction of equity. We show that this is not necessarily so. The intuition for our result is as follows. The VC demands a higher equity fraction because the investment made is assumed to be larger. A higher investment, however, produces a higher cash flow at time $t = 4$, leading the same fraction of equity to correspond now to a higher value. If the second effect dominates the first, we should have a decrease in the VC’s equity share. This happens if the average gain of the new investment is not much higher than the marginal gain of the new investment. In other words, for the second effect to dominate the first, it suffices that the investment in the new project, as a proportion of the investment done in the old projects, is high enough. Otherwise, the first effect dominates. Therefore, this ambiguity result can be seen as a consequence of assuming the coexistence of different projects.

4 Equilibrium in the public market

In this section we describe the public market and establish sufficient conditions supporting the decisions of firms to go public. We assume that a going-public firm (either good or bad) offers the investors a number $n$ of shares at a price $p$. Investors choose between participating with information, or not participating at all\(^8\).

Since all investors produce information, bad firms end up selling only a fraction of the offered shares. Let $N$ be the total number of investors in the market (with $N > n$). Only those who get wrong evaluations about bad firms will invest in those firms. The evaluation technology in (4) implies

\(^8\) Investors can not free ride on the information produced by others investors because, before the public offer, they cannot access the results of the evaluations made by other investors.
that a bad firm expects to sell \( n_B = \min(n, Ny) \) shares. Good firms get always good evaluations and hence they sell \( n \geq n_B \) shares. In particular, we assume this inequality to be strict, reflecting the fact that bad firms have more difficulties raising funds than good firms, leading to

\[
n_B = yN.
\]  

(11)

We now study the equilibrium relationships between the different parameters of the model.

First, the maximum price offered by firms must guarantee the participation of investors in the public offer, implying zero expected gains for investors. If informed investors had a positive expected gain, the firm could increase the selling price, lowering the investors’ expected gain to zero, increasing in this way the expected returns to insiders. Let \( m \) denote the number of shares hold by insiders. The zero expected gain condition reads

\[
(1 - \theta) \frac{1}{m + n} V_G + \theta y \frac{1}{m + n_B} k_B (D + pn_B) - [1 - (1 - y)\theta] p - c = 0. 
\]  

(12)

Notice that \( \theta \), the belief of each investor about the probability of the firm being an \textit{ex-ante} bad firm, is equal to the VC’s belief, as defined in equation (6).

Second, in order for investors to produce information, they must be compensated for the evaluation cost. This benefit results from not bidding a share in a bad firm, when investors get bad evaluations. Therefore we must have

\[
\theta(1 - y) \left[ p - \frac{1}{m + n_B} k_B (D + pn_B) \right] - c > 0. 
\]  

(13)

Third, we build the model such that investors have always advantage in producing information. Since the informed investors have zero expected gain, the uninformed investors must have a negative expected gain, leading to

\[
(1 - \theta) \frac{1}{m + n} V_G + \theta \frac{1}{m + n_B} k_B (D + pn_B) - p \leq 0.
\]

Fourth, good firms establish the price level \( p \) such that the expected amount raised is \( I \). With probability \( \phi_1 \), \textit{ex-ante} good firms are \textit{ex-post} good firms and sell all the offered shares. With probability \( (1 - \phi_1) \) \textit{ex-ante} good firms are \textit{ex-post} bad firms and sell only a fraction of the offered shares. It then follows that \( p \) and \( n \) are established such that

\[
\phi_1 pn + (1 - \phi_1)pn_B = I.
\]  

(14)
As a result, *ex-post* good firms obtain \( pn > I \), resulting in an amount exceeding the required investment. The excess funds, \( pn - I \), do not generate additional value to the firm. In contrast, the *ex-post* bad firm raises \( pm_B < I \), an insufficient amount to fully finance the investment.

This means that we have four equations (6,11,12) and (14) characterizing the equilibrium relationship among four unknown variables \((p, n, n_B, \theta)\). These relationships are characterized in the following proposition.

**Proposition 3 (Public Market Equilibrium)** There exists a price threshold \( p_L \), above which there is an equilibrium in the public market where the *ex-ante* good firm offers \( n^* \) shares at price \( p^* > p_L \), and the *ex-ante* bad firm pools with the *ex-ante* good firm. The *ex-post* good firm sells \( n^* \) shares, raising an amount of funds larger than \( I \). The *ex-post* bad firm sells \( n_B \) shares (less than \( n^* \)), raising an amount of funds lower than \( I \). All investors produce information and bid for a share if and only if they get a good evaluation. Such an equilibrium exists only if \( c < \bar{c} \) and \( k_B < \bar{k_B} \) (\( p_L, \bar{c} \) and \( \bar{k_B} \) are given in the appendix).

This equilibrium holds when bad firms are not too productive (\( k_B < \bar{k_B} \)), so that when the price increases, the investors face a decrease in gains. An increase in the offer price enables a firm to obtain a larger amount of funds when it ends the organization process as a bad firm. This corresponds to an increase in the investment done by that firm and in the investor’s expected gain. However, the investor has also to pay a higher price for the shares. Therefore, if a bad firm is not too productive, the investor loses when the price increases.

The out-of-equilibrium beliefs that support the above equilibrium are the following. Outsiders infer that any firm setting prices other than \( p^* \), or offering a number of shares other than \( n^* \) (at price \( p^* \)) is a bad firm *ex-ante* with probability 1. With these out-of-equilibrium beliefs, good and bad firms do not have advantage in deviating from the pooling equilibrium, because they are identified as *ex-ante* bad firms and have to sell a higher amount of shares at a lower price.

**Corollary 4 (Comparative Statics)** The equilibrium price \( p^* \) is (a) decreasing in the outsiders’ cost of information production; (b) increasing in the amount invested in the old firm’s projects and (c) increasing in the productivity of the *ex-post* good firm. The impact of the amount invested in new projects on the share price is ambiguous.

We now describe the economic intuition behind these results. The first result is simple to understand. When the cost of information production
increases, the investor’s expected gain decreases and becomes negative. This forces the firm to lower the offer price to ensure the investor’s participation in the public offer.

The second result is also quite intuitive. The higher the amount of funds invested in the past, other things equal, the higher will be the expected value of the firm, independently of his or her type. Someone investing in a firm that had large investments in the past will therefore have higher expected gains than individuals investing in firms with lower past levels of investment. Therefore, firms with higher level of past investments can sell the shares at a higher price, as compared to firms with lower level of past investments, other things equal.

Third, when an *ex-post* good firm becomes more productive, its expected value increases, causing an increase in the investor’s expected gain and in the share price.

Finally, an increase in the firm’s investment in new projects has two effects on the public market. First, *ex-post* good firms become more valuable, because they have more new projects that will produce more value in the future. However, *ex-post* bad firms do not take advantage of these opportunities, since they cannot raise additional funds to finance the required investment increment. Second, it happens that for the same price level, firms must sell more shares to raise the additional funds necessary to finance the new projects. This penalizes the outside investors, because the firm’s value is diluted among more investors. These two effects make the impact of an increase in $I$ in the investor’s expected gain ambiguous. This ambiguity contrasts with the result in Chemanur and Fulghieri (1999), who conclude that the impact of an increase of $I$ on the shares’ price is positive. However, under specific parameters’ specifications we can still clarify this ambiguity.

For instance, if the number of potential investors and/or the probability of wrong evaluations is large (implying $n_B$ closer to $n$), an increase in $I$ makes the share price to decrease.\(^9\) In the opposite situation, when $n_B$ is relatively small, we can have two cases. With $D$ small, firms with higher $I$ sell shares at a higher price.\(^{10}\) However, if $D$ is large, an increase in $I$ decreases the share price.\(^{11}\) This occurs because if $D$ is large, for the same price the firm needs to sell a larger number of shares to obtain a higher amount of funds. This has a strong negative effect on the investor’s expected gain. In such case the investor loses an important fraction of the old projects’ value, due to the increase in $n$.

\(^9\)See inequality (30) in Appendix 8
\(^{10}\)See Inequality (31) in the Appendix.
\(^{11}\)See Inequality (32) in the Appendix.
5 The Information Process

In the former sections we have characterized the equilibrium in the private equity market and the equilibrium in the public market. Given this knowledge, the management of the firm will prefer either one or the other, depending on the information generated by each of the alternatives. We hereby specify a model for the information process that will allow to choose the financing mode, according to the model parameters.

If the firm chooses to go public, the insiders obtain information from the amount of funds raised in the public offer. If the firm chooses the private equity market, the VC transmits to the firm the result of the evaluation performed by him or her. In the case of a public offer, the evaluation is made by a huge number of outsiders, reducing the uncertainty to zero and leading the insiders to know with certainty the ex-post type of the firm. In contrast, the information given by the VC to insiders the private equity market may be wrong with a given probability.

Once the active project attains the steady state, the role of the active investor comes to an end. Without additional information, the active investor’s rational decision is to leave the firm, because he or she thinks that the active project has been successful, to the best of his or her knowledge. However, there is a probability that the firm ends up as a bad firm, in which case the active investor should stay in the firm to redirect the project. Otherwise, the active investor may incur in costs such as, for example, a loss of reputation. Nevertheless, the active investor can get information during the financing process that conducts him or her to reconsider the initial decision.

The objective of the entrepreneur is not the same as the objective of the active investor. The entrepreneur is interested only in maximizing his or her fraction of the firm’s expected value or, in other words, in the best financing decision. The active investor, however, is not only interested in taking the best financing decision, but also in collecting information, so he or she can decide whether to exit the firm or not. Hence, the management gives more weight to the information as the weight of the active investor in the management board increases.

5.1 Information to insiders generated by the financing process

Before the financing process, outsiders are in disadvantage with respect to insiders in terms of information about the firm. Insiders know the ex-ante type, which allows to construct a belief about the ex-post type. However, insiders do not know their exact ex-post type. For instance, they may not know
Figure 1: Information to insiders generated by the public market

how much the competitors’ technology and the demand for their products has changed during the private period, making it impossible to evaluate their relative quality. In order to have a precise picture of these factors, insiders need to receive information from outsiders, since outsiders have advantage in collecting industry and markets’ specific information. In particular, in the case of LBOs, when the firm was public, the information collected by outsiders was transmitted to insiders namely throughout the share price. Once the firm is private, this information is transmitted in an imperfect way to insiders, generating uncertainty about the \textit{ex-post} type.

During the financing process, outsiders evaluate the firm and get information about its \textit{ex-post} type. This information is transferred to insiders, directly or indirectly, helping insiders to determine the \textit{ex-post} type of the firm. In what follows we analyze how the information is transmitted from outsiders to insiders.

If the firm chooses to go public an offer is made to the investors, incorporating the uncertainty about the \textit{ex-post} type. The amount of shares and their price are established such that the firm expects to raise the amount $I$ required to finance the new project. If the firm raises funds above $I$, insiders infer that the firm is good, since that is possible only if all investors had good evaluations. If the firm raises funds below $I$, insiders infer that the firm is bad, since this implies that some investors got bad evaluations and bad evaluations happen only if the firm is \textit{ex-post} bad. Hence, the insiders know with certainty the \textit{ex-post} type after a public offer, as illustrated by Figure 1.

In contrast, if the firm chooses the VC financing, the evaluation result
given by the VC to insiders is ambiguous as illustrated in Figure 2. If the VC gets a bad evaluation, insiders are sure that the firm bad. However, if the VC gets a good evaluation, insiders are not sure about the ex-post type, since there is the possibility that bad firms are evaluated as good firms.

5.2 The active investor’s decision and the financing process

In this Section we analyze how the active investor collects information during the financing process of the new project. Information about the ex-post type is relevant, since if the active investor decides to leave an ex-post bad firm, he or she may incur in costs, here denoted by \( C_R \). These costs may be due to a loss of reputation, if the market recognizes that the active investor was not efficient helping managing the active project; alternatively, \( C_R \) may be thought as the opportunity cost for the active investor, since it may be more profitable to extract additional results from the active project than to face the costs of initiating a new project elsewhere.

Consider the case of an an ex-ante good firm. We have three cases:

1. If the active investor does not collect information from the financing process, the decision to leave the firm leads to an expected loss

\[
(1 - \phi_1)C_R. \tag{15}
\]

2. If the firm chooses the public financing, the market gives the correct information about its ex-post type. Therefore, the active investor will
take always the correct decision and will never incur the cost $C_R$. Hence, the expected value\textsuperscript{12} of the collected information to the active investor is given by expression (15).

3. When the firm uses the VC, if the firm is really ex-post bad, the probability of getting a correct evaluation is $(1 - y)$. Therefore, the active investor’s expected gain of the collected information is\textsuperscript{13}

$$(1 - \phi_1)(1 - y)C_R.$$  

Finally, the expected gains from collecting information in ex-ante bad firm can be calculated in a similar way, simply replacing $\phi_1$ by $\phi_2$ in the above expressions.

5.3 The entrepreneur’s objective

The entrepreneur chooses the financing process that maximizes the value of the shares. Let $m_e$ denote the number of shares belonging to the entrepreneur and let $n_G$ ($n_B$) denote the number of sold shares when the firm is perceived as good (bad) by the market. Additionally, let $p$ denote the price in the public offer. If the ex-ante good firm chooses the public market financing, from equation (1) the expected gain for the entrepreneur at a given price $p$ is

$$
\phi_1 \frac{m_e}{m_e + n_G} k_G [D + \min(I, p m_G)] + (1 - \phi_1) \frac{m_e}{m_e + n_B} k_B [D + \min(I, p m_B)].
$$

When the VC is chosen to finance the project, the ex-ante good firm has also to consider two scenarios about its ex-post type. If the firm is good ex-post, then the VC gets a fraction of shares $s^*_g$ for sure, because the evaluation of the VC will always be good. However, if the firm is bad ex-post, the fraction that the VC gets depends on his or her evaluation. If the evaluation is good, the fraction is $s^*_g$, but if the evaluation is bad, the fraction is $s^*_b > s^*_g$. Hence, if the ex-ante good firm chooses the VC financing, from equations (3,2) the expected gain for the firm is

$$
\phi_1 (1 - s^*_g) k_G (D + I) + (1 - \phi_1) k_B (D + I) \left[ y (1 - s^*_g) + (1 - y) (1 - s^*_b) \right].
$$

Similarly, we can calculate the expected gain for an ex-ante bad firm, simply replacing $\phi_1$ by $\phi_2$ above.

\textsuperscript{12}This amount corresponds to the value that the active investor does not loose by taking the wrong decision of leaving the firm when the firm is bad.

\textsuperscript{13}Notice that, in this case, the firm combines its own private information ($\phi_1$) with the information given by the VC.
5.4 The objective of the management

The management is composed by the entrepreneur and the active investor. Therefore, the management’s decisions must take into account the interests of both actors.

The entrepreneur has a medium/long run commitment to the firm. Furthermore, the entrepreneur does not lose money in absolute terms by remaining in the firm, since the firm’s projects have a positive net present value independently of the actual firm’s ex-post quality. The objective of the entrepreneur is thus to maximize the expected value of his or her fraction of the firm and is not interested in the information collected in the financing process.

The active investor’s objective is to maximize his or her fraction of the firm’s expected value plus the expected gain from the information collected during the financing process. It follows that the objectives of the entrepreneur and active investor have a common element, i.e., the firm’s expected value, but differ in what refers to the value of information. We thus assume that the management’s objective depends on the degree of control that the active investor has in the management board. Let $\mu$ denote the influence that the active investor has in the management decisions, with $0 \leq \mu \leq 1$. If the management of an ex-ante good firm chooses to go public, its objective function is thus

$$\frac{\phi_1 m k_G}{m+n} [D + \min(I, pm_G)] + \frac{(1-\phi_1) m k_B}{m+n_B} [D + \min(I, pn_B)] + \mu (1-\phi_1) C_R,$$

where $m$ denote the number of shares belonging to insiders. When $\mu = 0$, the active investor has no influence in the management decisions, and the information collected in the financing process has no importance to the firm’s decisions. When $\mu = 1$, the active investor has a complete control over the management decisions and the collected information has the same importance in the management decisions as the fraction of the expected value that goes to insiders.

Similarly, if the management of an ex-ante good firm chooses the VC, then the expected value to the management is

$$\phi_1 (1-s_g^*) k_G (D + I) + (1-\phi_1) k_B (D + I) [y(1-s_g^*) + (1-y)(1-s_b^*)] + \mu (1-\phi_1)(1-y) C_R.$$

The management will choose the financing method that maximizes the relevant expected value.
6 Global equilibrium

In this section we describe and explore sufficient conditions that lead a firm in equilibrium to choose either a VC financing or to go public. The Proposition that follows describes that choice, and includes Propositions 1 and 3 that describe the equilibrium in the private equity market and in the public market. As stated in the next Proposition, the choice between the public market and the private equity market depends on the outsiders’ cost of evaluation.

**Proposition 5 (Global Equilibrium)** Let $\rho < \rho_P$, $c < \bar{c}$ and $k_B < \bar{k}_B$. There are numbers $c_G, c_B^{S_1}, c_B^{S_2}, \Psi_1$ and $\Psi_3$ such that (a) if $\Psi_1 > \Psi_3$ and $c_G < c < c_B^{S_1}$, bad firms choose to go public; (b) if $\Psi_1 < \Psi_3$ and $c_B^{S_2} < c < c_G$, bad firms choose VC financing. Away from these parameters’ constraints, both types of firms choose public financing if $c < c_G$, and choose VC financing if $c > c_G$. A sufficient condition for this equilibrium to exist is that the increase in the evaluation costs affects more adversely the firms’ gain with public financing than with VC financing (see equations (33, 34, 37) and (38) in the Appendix). The values of $c_G$, $\Psi_1$, $\Psi_3$, $c_B^{S_1}$, $c_B^{S_2}$, $\rho_P$, $\bar{c}$ and $\bar{k}_B$ are given in the Appendix.\(^{14}\)

If an increase in the evaluation costs decreases more the firms’ gain under public financing than under VC financing, then the above global equilibrium holds. That condition will be fulfilled easily because there are many outsiders producing information in the public market as compared to only one VC in the private equity market. The explanation for such fact is as follows. When the evaluation cost increases, the firm has to reduce the share’s price in the public market. In fact, with higher evaluation costs the investors are only willing to participate in the public offer if the firm sells its assets at a lower price. Since this reduction in the share price is multiplied by many investors, this implies a high reduction in the firm’s expected gain. In other words, the firm has to compensate each investor for the increase in the evaluation costs. In the private equity market, however, only the VC evaluates the firm. If the evaluation cost increases, the firm only has to compensate the VC for this increase. As a consequence, the global reduction in the firm’s expected gain is smaller in the VC financing than in the public financing.

It also follows from the above argument that firms go public when the costs of evaluating them are sufficiently low, in agreement with the main result stated in the proposition.

\(^{14}\)Out-of-equilibrium beliefs: If the outsiders observe a firm going public with $c > c_G$ or utilizing the VC financing with $c < c_G$, they believe that the firm is *ex-ante* bad with probability 1.
Generally speaking, our result point to a pooling equilibrium. Under certain market parameters’ values, however, and as opposed to what happens in Chenmanur and Fulghieri (1999), good and bad firms separate their behaviors. Let us discuss some of the intuition underlying these different circumstances.

In our pooling equilibrium, good firms choose the better financing alternative for themselves, and bad firms follow this choice in order to sell their shares at a high price. Good firms do always what is better for themselves because the market’s beliefs are aligned with their actions. By contrast, bad firms cannot do always what is better for themselves because, in some situations, they can be identified as bad \textit{ex-ante}.

However, under some specific market parameters firms separate their behavior. In particular, when the evaluation cost is between \( c_{B}^{S2} \) and \( c_{G} \), good firms continue to go public and bad firms may prefer the VC financing. This occurs because, under these parameters, bad firms expect higher gains with a VC financing than with public financing, even being identified \textit{ex-ante} as a bad firm. This separating preference of bad firms happens under two main conditions. First, if the public market is more sensitive than the VC to changes in the probability that firms are \textit{ex-post} good (i.e., \( \Psi_1 - \Psi_3 < 0 \)). In other words, the public market penalizes \textit{ex-ante} bad firms more than the VC, in terms of expected gains. Second, the likelihood of this equilibrium is higher when the probability of becoming an \textit{ex-post} good firm is not much lower for an \textit{ex-ante} bad firm (\( \phi_2 \)) than for an \textit{ex-ante} good firm (\( \phi_1 \)). This means that the active project may actually transform \textit{ex-ante} bad firms in \textit{ex-post} good firms. In sum, for the assumed range of evaluation costs, the two conditions above justify a separating strategy for the bad firm.

In the remaining of this section we describe how changes in some parameters may affect the decision of going public. We begin by analyzing the impact of the active investor’s degree of control over the firm.

**Corollary 6 (Active Investor’s Degree of Control)** The \textit{critical value of the evaluation costs preventing firms to go public increases with the active investor’s degree of control}.

Since the active investor values more than the entrepreneur any information that allows to identify the firm’s type, as the active investor’s degree of control \( \mu \) increases, the firm increases its preference for the public financing, as expressed in its objective functions (16) and (17), and goes public for higher evaluation costs.

---

\(^{15}\)See inequality (39) in Appendix 8.
This result is consistent with the empirical evidence of Jelic, Saadouni and Wright (2001) if we assume that longer private periods lead to lower evaluation costs. Their study of 167 MBOs in the London Stock Exchange points out that venture backed MBOs tend to go public earlier than non-venture backed MBOs.

Next, we study the impact of the public market dimension on the decision of going public.

**Corollary 7 (Potential Investors)** *The critical value of the evaluation costs preventing firms to go public increases with the number of potential investors in the public market.*

The public market dimension $N$ may restrict the performance of an *ex-post* bad firm, since only a fraction of the total number of investors will buy its shares. Hence, if $N$ is not large enough, the firm may not raise the required amount to invest in the new project and may loses value.

As the number of potential investors $N$ increases, an *ex-post* bad firm may sell a larger number of shares, raising a higher amount of funds to invest. From the point of view of the outside investors, on one hand it is good that the investment is higher, since the firm becomes more valuable; on the other hand, selling a larger number of shares dilutes the ownership of the firm. The dilution effect is particularly strong when the investments done in the past ($D$) are large, because large $D$ should imply large expected gains. However, the first effect proves to be stronger than the second effect, and the expected gains for investors resulting from *ex-post* bad firms increase when the number of investors in the market increases.

When the number of potential investors increase, the expected gains for investors also increase if they invest in *ex-post* good firms. This increase comes from the fact that, with a larger number of investors, firms can establish, for the same number of shares, a lower price to get the amount $I$. This benefits the investors because they can buy shares at a lower price.

Hence, the total expected gains for investors increase with $N$, since the expected gain when they invest in *ex-post* good or bad firms increases. This enables firms to increase the price of shares as $N$ increases, selling a lower number of shares. Let us see the impact of the changes in $p$ and $n$ in the firms’ expected gain. On one hand, when the firm is *ex-post* good, it needs to sell less shares to obtain the same amount to invest, implying that *ex-post* good firms obtain $I$ giving up a smaller fraction of equity. On the other hand, when the firm is bad *ex-post*, the increase in $N$ has two effects of opposite signal. First, the firm sells more shares at a higher price, the amount of funds raised increases, and insiders have a higher expected gain. Second, selling
more shares dilutes the insiders’ ownership, diminishing the expected gains to insiders. The joint result of these effects is that the firm’s expected gain increases.

Notice that both *ex-ante* good and *ex-ante* bad firms do not know their real type before the financing process starts. Therefore, both *ex-ante* types of firms increase their expected gains going public when the market dimension increases. This occurs because, as the market increases, the financing conditions in the public market improve to the *ex-post* good and to the *ex-post* bad firms, but do not change in the private equity market. This implies that in larger public markets firms go public for higher evaluation costs.

This point is somehow illustrated by the empirical findings for IPOs of Zingales (1998) and Rydqvist and Hogholm (1994) for European firms, and Gompers (1993) and Lerner (1994) for US firms. These authors provide evidence that, on average, European companies go public later than US companies. In the context of our model, an explanation for this fact may be the larger dimension of the US stock market with respect to the European stock market. In fact, in 2002 the stock market capitalization was 47% of the GDP in the Euro area, and 104% of the GDP in the United States\textsuperscript{16}.

Notice that our explanation is slightly different from the one presented in Chenmanur and Fulghieri (1999). These authors argue that the difference in the age of firms that go public between Europe and the USA is due to the fact that in Europe there are less financial intermediaries producing information about firms. This implies that, in general, firms have higher evaluation costs in Europe than in the USA. Our claim is that the observed difference between the average age of European and USA firms that go public is due to the lower dimension of the European stock market. Of course both arguments are strongly related since the number of financial intermediaries can be seen as a good proxy of the market dimension.

The next paragraphs analyze the impact of the dimension of the old and new firm’s projects and of the firm’s productivity in the decision of reverting the LBO.

**Remark 8** In generic terms, the impact of changes in $I$, $D$ or $k_G$ on the decision of reverting the LBO is ambiguous. These impacts depend on the values of the model’s parameters.

The impact of a change in $I$ on the decision of going public is ambiguous. Let us consider two mutually exclusive situations. First, if increasing $I$ leads the selling price to increase, then (a) the expected gains of the firm under a public offer will increase and (b) the expected gains under the VC financing

\textsuperscript{16}Data available from the World Federation of Exchanges.
may also increase. However, it is not possible to say which benefit is preferable. Second, if increasing $I$ leads the selling price to decrease, then it is simply impossible to characterize, in general, what happens to the expected gains of the firm. Let us describe these distinct situations.

From Corollary 4 we know that a sufficient condition for share price to increase as $I$ increases is that (a) an \textit{ex-post} bad firm sells a relatively small number of shares and (b) the old projects are relatively small as compared to the dimension of the new projects. If share prices increase due to an increase in $I$, the following happens: (a) \textit{ex-post} good firms have a larger expected value because invest more ($I$ increases), diluting less the equity ($n$ decreases); and (b) \textit{ex-post} bad firms have more resources to invest ($pm_B$ increases) selling the same number of shares ($n_B$ is constant) at a higher price. Hence, as the investment in new projects increases there is an increase in the firms’ expected gain with public financing, because the value accruing to the firms increase, no matter what their real type is.

Consider now the case where the share price decreases as $I$ increases. In this case, the increase in $I$ has an ambiguous impact on the firms’ expected gain with public financing. This can be analyzed as follows (a) the \textit{ex-post} bad firm invests less because the selling price decreased, lowering its expected value; (b) the \textit{ex-post} good firm sells more shares, diluting more its equity and decreasing its expected value, but simultaneously invests more in new projects, increasing the firm’s expected gain.

On the private equity market, an increase in $I$ demands more funds from the VC. The VC demands in exchange a higher equity’s fraction. With new projects of higher dimension the firm has a higher expected value. This implies that the same fraction of equity corresponds now to a higher VC’s remuneration. These two effects together make the impact of an increase in $I$ on the fraction to the VC ambiguous. However, we may eliminate this ambiguity if the investment in new projects ($I$) is high enough as compared to the investment in old projects ($D$). In this case\textsuperscript{17}, an increase in $I$ decreases the fraction to the VC and increases the expected gain with VC financing.

In conclusion, we cannot say generically what is the effect of $I$ on the decision of going public. First, we do not know the exact impact of $I$ in the firms’ expected gain with each financing method. Second, even when the change in $I$ increases the firm’s gain with both financing methods (which is likely to occur for small $D$), we do not know which financing method provides the largest increase of gains.

We next examine the case of a change in $D$. The expected value of the firms increase with both private and VC financing but, once again, it is not

\textsuperscript{17}See Appendix.
possible to ensure which alternative works best for the firm.

From Corollary 4 an increase in $D$ increases $p^*$. This enables the firm to sell less shares at a higher price than before the increase in $D$. Consequently, the \textit{ex-post} good firm benefits from a larger amount of funds invested in old projects and also from the fact that it sells less shares, diluting less its equity. The \textit{ex-post} bad firm also benefits from the higher funds invested in old projects and from the higher price at which can sell shares, allowing for a higher investment in new projects. This implies that the expected gain of both types of firms increases when they go public with higher $D$.

On the private equity market, since firms with large $D$ have a higher value than firms with low $D$, the VC demands a lower fraction of equity from the former. Therefore, firms with large $D$ dilute less more valuable equity, implying an increase in their expected gain when financed by the VC.

Since we cannot establish under what type of financing the expected gains of the firms increase more, we cannot conclude on the impact of $D$ in the decision of going public. However, if it is possible to say (for given parameters) that an increase in $D$ leads the firms’ expected gain to increase more with public financing than with VC financing, then firms with higher $D$ should go public for higher evaluation costs.

The impact of $k_G$ is very similar to the impact of $D$. An increase in the good firm’s productivity implies a higher investor’s expected gain and a higher share price. The \textit{ex-post} good firm has a higher expected value because the projects have a higher productivity and the firm needs to sell less shares to obtain the amount $I$. An increase in $k_G$ also allows the \textit{ex-post} bad firm to invest more in new projects, since it may sell shares at a higher price, increasing its expected value. Therefore, both types of firms increase their expected gain under the going public decision, as $k_G$ increases. On the private equity market, an increase in $k_G$ leads to a reduction of the VC’s share of equity and to an increase of the firm’s expected gain. As before, it is not possible to conclude about the impact of $k_G$ in the decision of going public.

\textbf{Corollary 9 (Investment’s Risk)} \textit{The critical value of the evaluation costs preventing firms to go public increases as the investments’ risk or the covariance between the cash flows of the old and new projects increases.}

Regarding the first part of the statement, notice that the risk-averse VC demands a higher risk premium when faced with firms with higher investment uncertainty (for example, resulting from higher technological uncertainty). This makes the VC less attractive as compared to the public financing alternative. For the same evaluation costs, the attractiveness of the public
financing increases with the investment uncertainty. This implies the state-
ment above.

Regarding the second part of the statement just notice that a higher
covariance between the cash flows of the old and new projects of a firm
implies a higher risk of the total cash flows. A similar argument based on
the risk aversion of the VC leads to the second part of the statement.

Next we focus on the long-run performance of public offers as compared
to their comparison firms. Actually, a number of empirical studies such as
DeGeorge and Zeckhauser (1993), Mian and Rosenfeld (1993) and Holthausen
and Larker (1996), among others, support the fact that reverse LBOs in the
long-run tend to out-perform the market. In the case of IPOs, strong evidence
can be found in Gompers and Lerner (2001) and Ma and Shen (2003), among
others, that they do not perform above the market. We hereby characterize,
in the context of our model, situations that are compatible with such different
types of evidence.

Recall that before the active project began in the private period, the pro-
portion of \textit{ex-ante} good firms is \(\phi\). After the conclusion of the reorganization
process, when firms decide to go public or not, the proportion of good firms
\textit{(ex-post)} in the group of firms that have made the public offer is \(1 - \theta\). In
contrast, we assume that the proportion \(\phi_M\) of good firms already listed in
the stock market remains constant across time. We assume also that the
proportion of good firms in the group of private firms is higher after the
organization than before the organization, \textit{i.e.,} \(1 - \theta > \phi\). In other words,
firms improve their quality, on average, during the active project. This im-
plies that the fraction of \textit{ex-ante} good firms that remains \textit{ex-post} good firms
is higher than the fraction of good firms in the group of private firms, \textit{i.e.,}
\(\phi_1 > \phi\). Additionally, we assume that, once the active project attains the
steady state, the costs of evaluation of both good and bad firms are distrib-
uted uniformly in the interval \([0, \overline{c}]\), where the equilibrium in Proposition 5
exists.

Consider the two possible pooling equilibria described in Proposition 5.
Under that Proposition, we characterize two mutually exclusive situations
depending on the values of the parameters. Situation 1 is defined by values
of parameters such that \(\Psi_1 > \Psi_3\) for \(c = c_G, \rho = \rho_1\), whereas situation 2 is
defined by \(\Psi_1 < \Psi_3\) for \(c = c_G, \rho = \rho_1\).

Consider now a pooling equilibrium under situation 1. There is a propor-
tion \((c_G/\overline{c}) \phi\) of good firms that go public and a proportion \((c_G/\overline{c}) (1 - \phi)\) of
bad firms that go public. The proportion of \textit{ex-post} good firms in the set of
public firms is

\[
\frac{cC\phi\phi_1 + cC (1 - \phi) \phi_2}{cC} = \phi\phi_1 + (1 - \phi) \phi_2.
\]

We characterize the conditions under which the proportion of ex-post good firms in the group of public offers is larger than the proportion of good firms in the market (\(\phi_M\)). In other words, we want to determine under what circumstances \(\phi\phi_1 + (1 - \phi) \phi_2 > \phi_M\).

First, let us analyze the extreme cases. On one hand, if ex-ante bad firms become ex-post good firms, i.e., \(\phi_2 > \phi_M\), then the proportion of ex-post good firms in the group of public offers is always higher than the proportion of good firms in the market. On the other hand, if ex-ante good firms become ex-post bad firms, i.e., \(\phi_1 < \phi_M\), then the proportion of public offers that are ex-post good firms is always lower than the proportion of good firms in the market.

Outside these extreme cases, the relation between \(\phi\phi_1 + (1 - \phi) \phi_2\) and \(\phi_M\) depends on the relation between \(\phi\) and \(\phi_M\). If \(\phi \geq \phi_M\), then \(\phi\phi_1 + (1 - \phi) \phi_2 > \phi_M\). This occurs because we assume that \(1 - \theta > \phi\). However, if \(\phi < \phi_M\), then the inequality \(\phi\phi_1 + (1 - \phi) \phi_2 > \phi_M\) is only verified if

\[
\phi > \Phi \equiv \frac{\phi_M - \phi_2}{\phi_1 - \phi_2},
\]

where \(\Phi < \phi_M\).

This explanation refers to Situation 1 with pooling equilibrium. In Situation 1 with separating equilibrium, the conclusions are similar, but we cannot ensure that \(\Phi < \phi_M\), because the proportion of ex-ante bad firms going public is larger than the proportion of ex-ante good firms going public.

These results can be extended to situation 2 described above. With pooling equilibrium the conclusions are similar to those above, with the difference that \(c_B\) must be used in place of \(c_G\). With a separating equilibrium the conclusions are similar, but the cutoff parameter is lower than \(\Phi\) defined in (18). This occurs because the ex-ante good firms go public for higher costs than the ex-ante bad firms.

Therefore, we can state the following Proposition.

**Proposition 10 (Average Quality of Public Offers)** If \(\phi_2 > \phi_M\), then the proportion of ex-post good firms in the group of public offers is higher than the proportion of good firms in the market. If \(\phi_1 < \phi_M\), there will never exist in the group of public offers a proportion of ex-post good firms as high as the proportion of good firms in the market. If \(\phi_1 \geq \phi_M \geq \phi_2\) and \(\phi > \Phi\),

\[
\text{27}
\]
the proportion of ex-post good firms in the group of public offers is higher than the proportion of good firms in the market. Furthermore, we can always ensure that $\Phi < \phi_M$ except in the case of a separating equilibrium in Situation 1 as characterized above.

In general, the proportion of ex-post good firm in the group of public offers is higher than the proportion of good firms in the market (as stated in Proposition 10). Therefore, we expect that the public offers, after going public, present an average performance above the average market’s performance.

The fact driving the positive abnormal performance of the public offers is that firms decide to go public not taking into account the information about their ex-ante types. Since the equilibrium considered is pooling, the proportion of ex-ante good firms going public is equal to the proportion of ex-ante bad firms going public. However, on average, the quality of firms improve after the active project. In fact, the proportion of ex-post good firms in the set of private firms is higher than the ex-ante good firms. This implies that the proportion of ex-post good firms is higher in the set of public offers than in the set of private firms. Therefore, if the percentage of ex-ante good firms was not much smaller than the percentage of good firms in the market, then the proportion of ex-post good firms in the group of public offers is higher than that proportion in the market.

7 Conclusion

In our model we consider firms that are not aware of their type after a private period. We conclude that a firm goes public only if the cost of its evaluation is sufficiently low. This occurs because an increase in the evaluation costs affects more adversely the firm’s gain under the public financing than under the VC financing, since there are many outsiders producing information in the public market and there is only the VC in the private equity market. This result is similar to the conclusion attained by Chemmanur and Fulghieri (1999), in the case where firms knew their type.

Generally, both types of firms choose the same financing source. However, and as opposed to the results of Chemmanur and Fulghieri (1999), in some circumstances, and for some intervals of evaluation costs, the firms may have separating behaviors. Namely, we can have a situation where good firms go public for higher evaluation costs than bad firms. This occurs when the public market is more sensitive than the VC to variations in the probability of the firm to be good ex-post.
We next describe the impact of different factors in the going public decision.

1. *The active investor*. The active investor wishes to leave the firm after the active project attains a steady state, only if the project has been well succeeded. This investor can get a more accurate information about the firm’s type in the public market than in the private equity market, since the public offer aggregates firm’s evaluations from a large set of investors. The entrepreneur does not use the information collected in the financing process. Hence, as the weight of the active investor in the management increases, the easier it is for the firm to go public.

2. *Public market dimension*. An increase in the market dimension increases the investor’s expected gain, because enables the *ex-post* bad firm to invest more and the *ex-post* good firm can establish, for the same number of shares, a lower price to get the amount $I$. With investors earning more, firms can establish a higher price and sell less shares in equilibrium. A larger price, together with an increase in the investment by the *ex-post* bad firm, implies that both firms’ expected gains increase when they go public but remain the same if they choose the VC.

3. *Investment in new projects*. Investing more in new projects has two effects: increases the firm’s expected value and demands additional outflow from outside investors, including the VC. These two opposite effects and the fact that $D > 0$, makes the impact of an increase in $I$ on the investors’ gain and on the VC’s utility ambiguous. This implies that, as opposed to Chemmanur and Fulghieri (1999), who considered the case $D = 0$, we cannot say what is the overall impact of a new project’s dimension on the share price and on the equity’s share to the VC. Therefore, it is not possible to know the impact of an increase in $I$ on the firm’s gain when it chooses the VC financing or the public financing.

4. *Investment in old projects*. A firm with more funds invested in old projects has a higher value without the need of additional investment by the current investors. On one hand, this causes an increase on the share price and in the firm’s expected gain with public financing. On the other hand there is a decrease in the equity’s share to the VC and an increase in the firm’s expected gain when it chooses the private equity market. Therefore, we cannot conclude on the general impact
of $D$ in the going public decision if the parameters of the economy are unknown.

5. **Firm’s productivity.** The impact of $k_G$ is very similar to the impact of $D$. An increase in the *ex-post* good firm productivity implies a higher investor’s expected gain, a higher share price, and a higher expected gain to the firm in the public market. On the private equity market, an increase in $k_G$ decreases the share of equity to the VC and the expected gain to the firm increases. Again, it is not possible to make a general statement about the impact of $k_G$ in the going public decision.

Our model provides an explanation for the fact that public offers, in particular reverse LBOs, may present a long-run performance above the one of their comparison firms. To start with, notice that, in the context of our model, firms decide to go public taking into account their *ex-ante* types. This implies that the proportion of good firms going public is equal to the proportion of bad firms going public. However, the proportion of *ex-post* good firms is higher than the proportion of ex-ante bad firms. This implies that the proportion of *ex-post* good firms in the set of public offers is higher than in the set of private firms. Therefore, if the proportion of good firms in the set of private firms, just before the organization, is not much smaller than the proportion of good firms in the market, then the proportion of *ex-post* good firms in the group of public offers is higher than in the market.

Finally we discuss one of the disadvantages of the VC, as compared to the public market, namely that the VC evaluates the firm only once. This evaluation is not confirmed by other evaluators and the VC may give a wrong signal to insiders. However, the VC can at least partially compensate this disadvantage with a better evaluation technology. In other words, the probability of identifying a bad firm as a good one is lower when the evaluation is made by the VC than when it is made by an isolated investor from the public market. Hence, the VC can offer better financing conditions to the firms with good evaluations. This implies that the expected gain to the firm with VC financing increases with respect to the public financing. An increase in the VC capacity of evaluating firms should therefore imply that the critical value of the evaluation costs that prevents the firms to go public must decrease. At this stage, this statement represents a conjecture to be proven in further research.
8 Appendix

8.1 Proof of Proposition 1

First we determine $s_b^*(c, \rho)$, the equilibrium fraction of equity to the VC.

If $e = b$, then $s_b^*(c, \rho)$ is the smallest solution of equation (8). Define
\[ \rho_1 \equiv \frac{V_e - I - c}{\sigma_g^2} \] and take \( \rho < \rho_1 \Rightarrow s_b^* < 1 \). If $e = g$, then $s_g^*(c, \rho)$ is the smallest solution of equation (7). Define \( \rho_2 \equiv \frac{V_g - I - c}{\sigma_g^2 + \sigma_b^2} \) and take \( \rho < \rho_2 \Rightarrow s_g^* < 1 \). Now we define \( \rho_p \equiv \min(\rho_1, \rho_2) \).

Next, we prove that firms prefer the pooling contract to the separating contract. The expected gain of the good firm in the pooling contract is given by equation (17) and in the separating contract is
\[
\phi_1 (1 - s_{gb}^*) k_G (D + I) + (1 - \phi_1) [g (1 - s_{gb}^*) k_B (D + I) + (1 - y) (1 - s_{gb}^*) k_B (D + I)] + \mu (1 - \phi_1) (1 - y) C_R,
\]
where $s_{gb}^*$ and $s_{gb}^*$ are given by equations (9) and (10) respectively.

The expected gain of the bad firm in the pooling contract is given by
\[
\phi_2 (1 - s_g^*) k_G (D + I) + (1 - \phi_2) [g (1 - s_g^*) k_B (D + I) + (1 - y) (1 - s_g^*) k_B (D + I)] + \mu (1 - \phi_2) (1 - y) C_R.
\]
and in the separating contract is
\[
\phi_2 (1 - s_{gb}^*) k_G (D + I) + (1 - \phi_2) [g (1 - s_{gb}^*) k_B (D + I) + (1 - y) (1 - s_{gb}^*) k_B (D + I)] + \mu (1 - \phi_2) (1 - y) C_R.
\]
In order to compare the expected gains above, we first show that $s_g^* < s_{gb}^*$ and $s_b^* = s_{gb}^*$.

**Lemma 11** $s_g^* < s_{gb}^*$ and $s_b^* = s_{gb}^*$.

**Proof.** $V_g > V_{gb}$ and $\sigma_{gb}^2 > \sigma_g^2$ implies $s_g^* < s_{gb}^*$ by definition. Notice that
\[
V_g = \Pr(q = G \mid e = g) k_G I + [1 - \Pr(q = G \mid e = g)] k_B I
\]
\[
V_{gb} = \Pr(q = G \mid e = g, s_{gb}^*) k_G I + (1 - \Pr(q = G \mid e = g, s_{gb}^*)) k_B I.
\]
Since $k_G > k_B$

\[
V_g > V_{gB} \iff \Pr(q = G \mid e = g) > \Pr(q = G \mid e = g, s^*_{gB}) \quad \iff \quad \frac{1 - \theta}{(1 - \theta) + y\theta} > \frac{\phi_2}{\phi_2 + y(1 - \phi_2)}.
\]

(19)

Since $\phi_1 > \phi_2$ it follows that $V_g > V_{gB}$. On the other hand,

\[
\sigma^2_g = \Pr(q = G \mid e = g) [(V_g - k_G I)^2 - (V_g - k_B I)^2] + (V_g - k_B I)^2
\]

\[
\sigma^2_{gB} = \Pr(q = G \mid e = g, s^*_{gB}) [(V_{gB} - k_G I)^2 - (V_{gB} - k_B I)^2] + (V_{gB} - k_B I)^2.
\]

The assumption that $\Pr(q = G \mid e = g, s^*_{gB}) > 0.5$ leads to $\sigma^2_{gB} > \sigma^2_g$ implying $s^*_g < s^*_{gB}$.

When $e = g$, we have $s^*_b = s^*_{gB}$, because $\Pr(q = G \mid e = b) = \Pr(q = G \mid e = b, Bad) = 0$ and $V_{gB} = V_b$.

Comparing the expected gains to the good and bad firms under the two types of contracts, both types of firm prefer the pooling contract resulting in a pooling equilibrium.

### 8.2 Proof of Corollary 2

Define

\[
F(s_g) \equiv s_g V_g - \rho s^2_g (\sigma^2_g + \sigma^2_{\varepsilon D} + \sigma^2_{\varepsilon I} + 2\sigma_{DL}) - I - c
\]

\[
T(s_b) \equiv s_b V_b - \rho s^2_b (\sigma^2_{\varepsilon D} + \sigma^2_{\varepsilon I} + 2\sigma_{DL}) - I - c.
\]

and notice that $F(s^*_g) = T(s^*_b) = 0$. We assume that utilities are increasing in wealth, meaning that $\frac{\partial F}{\partial s_g} > 0$ and $\frac{\partial T}{\partial s_b} > 0$. We thus have

\[
\frac{\partial s^*_g}{\partial c} = -\frac{\partial F}{\partial s_g} = -\frac{1}{\frac{\partial F}{\partial s^*_g}} > 0
\]

(20)

and

\[
\frac{\partial s^*_b}{\partial c} = -\frac{\partial T}{\partial s_b} = -\frac{1}{\frac{\partial T}{\partial s^*_b}} > 0.
\]

(21)

Let us prove that an increase in $\rho$ increases $s^*_g$ and $s^*_b$. In fact, we have

\[
\frac{\partial s^*_b}{\partial \rho} = \frac{\partial T}{\partial s_b} = s^*_b (\sigma^2_{\varepsilon D} + \sigma^2_{\varepsilon I} + 2\sigma_{DL}) > 0
\]

\[
\frac{\partial s^*_g}{\partial \rho} = \frac{\partial F}{\partial s_g} = \sigma^2_g > 0.
\]

32
and
\[ \frac{\partial s^*_g}{\partial \rho} = -\frac{\partial F}{\partial \rho} = s^*_g (\sigma^2_{\sigma_d} + \sigma^2_{\epsilon_I} + 2\sigma_{DI}) > 0. \] (22)

An increase in \( \sigma^2_{\epsilon_D} \) and \( \sigma^2_{\epsilon_I} \) increases \( s^*_g \) and \( s^*_b \) because
\[ \frac{\partial s^*_b}{\partial \sigma^2_{\epsilon_D}} = \frac{\partial s^*_b}{\partial \sigma^2_{\epsilon_I}} = \frac{\rho s^2_b}{\partial \sigma^2_B} > 0 \]
and
\[ \frac{\partial s^*_g}{\partial \sigma^2_{\epsilon_D}} = \frac{\partial s^*_g}{\partial \sigma^2_{\epsilon_I}} = \frac{\rho s^2_g}{\partial \sigma^2_B} > 0. \]

The fractions \( s^*_b \) and \( s^*_g \) are increasing with \( \sigma_{DI} \) since
\[ \frac{\partial s^*_b}{\partial \sigma_{DI}} = -\frac{2\rho s^2_b}{\partial \sigma_{B}} > 0 \]
and
\[ \frac{\partial s^*_g}{\partial \sigma_{DI}} = -\frac{2\rho s^2_g}{\partial \sigma_{B}} > 0. \]

However, \( I \) has an ambiguous impact on \( s^*_g \) and \( s^*_b \). Calculating
\[ \frac{\partial s^*_b}{\partial I} = \frac{\partial s^*_b}{\partial s^*_f} - \frac{s^*_b k_B - 1}{\frac{\partial I}{\partial s^*_f}}, \]
from (8) we have
\[ s^*_b k_B \left( \frac{D + I}{I} \right) - 1 = \rho s^*_b^2 \left( \frac{\sigma^2_{\sigma_d} + \sigma^2_{\epsilon_I} + 2\sigma_{DI}}{I} \right) > 0. \]

If the difference between \( k_B = \frac{\partial V_g}{\partial I} \) and \( k_B \left( \frac{D + I}{I} \right) = \frac{V_B}{I} \) is small (i.e., \( (D + I)/I \) is not too large), then we can ensure that
\[ s^*_b k_B - 1 > 0, \] (23)
implying \( \partial s^*_b / \partial I < 0. \)

With respect to \( s^*_g \) we get
\[ \frac{\partial s^*_g}{\partial I} = -\frac{\partial F}{\partial s^*_g} = -\frac{s^*_g \partial V_g}{\partial s^*_g} - 1, \]
with
\[ V_g = k_G(D + I) \Pr(q = G|e = g) + k_B(D + I) [1 - \Pr(q = G|e = g)] . \]

This implies that
\[ \frac{\partial V_g}{\partial I} = k_G \Pr(q = G|e = g) + k_B [1 - \Pr(q = G|e = g)]. \]
and
\[ \frac{V_g}{I} = k_G \left( \frac{D + I}{I} \right) \Pr(q = G|e = g) + k_B \left( \frac{D + I}{I} \right) \left[ 1 - \Pr(q = G|e = g) \right]. \]
Thus,
\[ \frac{V_g}{I} = \left( \frac{D + I}{I} \right) \frac{\partial V_g}{\partial I}. \]
We have from expression (7) that
\[ s^*_g \frac{V_g}{I} - 1 = \rho s^2_g \left[ \sigma^2_g + \left( \sigma^2_{zD} + \sigma^2_{zI} + 2\sigma_{DIP} \right) \right] \frac{I}{I} + c > 0. \]
Therefore, if \( \partial V_g/\partial I \) is not much smaller then \( V_g/I \) (i.e., \( (D + I) / I \) is not too large), we can guarantee that
\[ s^*_g \frac{\partial V_g}{\partial I} - 1 > 0. \quad (24) \]
This implies that \( \partial s^*_g / \partial I < 0 \).
An increase in \( D \) decreases \( s^*_g \) and \( s^*_b \) since
\[ \frac{\partial s^*_b}{\partial D} = -\frac{\partial F}{\partial \sigma^*_b} = -s^*_b \frac{\partial V_g}{\partial \sigma^*_b} < 0, \]
\[ \frac{\partial s^*_g}{\partial D} = -\frac{\partial F}{\partial \sigma^*_g} = -s^*_g \frac{\partial V_g}{\partial \sigma^*_g} < 0. \]
Finally, the equity fractions \( s^*_g \) and \( s^*_b \) are decreasing with \( k_G \) because
\[ \frac{\partial s^*_g}{\partial k_G} = -\frac{\partial F}{\partial k_G} = -s^*_g \frac{\partial V_g}{\partial k_G} < 0, \]
\[ \frac{\partial s^*_b}{\partial k_B} = -\frac{\partial F}{\partial k_B} = -s^*_b \frac{D + I}{\partial k_B} < 0. \]

8.3 Proof of Proposition 3

Let us prove that the system of equations characterizing the going public decision has at least a solution as described in the Proposition. If the system has multiple solutions, the one with largest \( p \) is chosen by the firm.

First, we characterize the threshold \( p_L \) as the price that implies a zero expected gain to the uninformed investor and prove that \( p^* > p_L \) whenever equilibrium exists. In order to prove this partial result notice that, if
equilibrium exists, the equilibrium price \( p^* \) satisfies equations (14) and (12). Isolating \( n \) from the former and replacing it in the latter, it follows that \( p^* \) is the root of \( F(p) = 0 \) where

\[
F(p) \equiv \frac{p(1 - \theta)\phi_1 V_G}{p[\phi_1 m - (1 - \phi_1)n_B] + I} + \frac{\theta y k_B (D + n_B p)}{m + n_B} p - [1 - (1 - y)\theta] p - c.
\]

Now let

\[
p_{\text{max}} = \arg \max \left\{ \frac{p(1 - \theta)\phi_1 V_G}{\{p[\phi_1 m - (1 - \phi_1)n_B] + I\}^2} \right\}, \text{ with } 0 < p < \frac{I}{(1 - \phi_1)n_B}.
\]

We now define \( k_B^* \) as the value of \( k_B \) such that \( F(p_{\text{max}}) = 0 \).

**Lemma 12** If equilibrium exists and \( k_B < k_B^* \), then \( p^* > p_L \).

**Proof.** Since

\[
\frac{\partial F(p)}{\partial p} = \frac{(1 - \theta)\phi_1 V_G I}{\{p[\phi_1 m - (1 - \phi_1)n_B] + I\}^2} + \frac{\theta y n_B k_B}{m + n_B} - [1 - (1 - y)\theta],
\]

we have

\[
\frac{\partial F(p)}{\partial p} < 0 \text{ for all } p \in \left(0, \frac{I}{(1 - \phi_1)n_B}\right).
\]

for all \( k_B < k_B^* \). Now, let us characterize \( p_L \). Isolating \( n \) in equation (14) we may write the expected gain to the uninformed investor as

\[
G(p) = \frac{(1 - \theta)\phi_1 V_G p}{p[\phi_1 m - (1 - \phi_1)n_B] + I} + \frac{\theta k_B (D + pn_B)}{m + n_B} - p,
\]

and define \( p^L \) as the root of \( G(p) = 0 \). From the equilibrium condition (13) it follows that \( F(p) > G(p) \) for all \( p \). In particular \( F(p^L) > G(p^L) = 0 \). This fact, together with \( F(p^*) = 0 \) and inequality (26) implies that \( p^* > p^L \). ■

We next characterize a sufficient upper bound on the cost of information for equilibrium to exist. Let \( p|_{\text{H}}(n) \) denote the price satisfying equation (14) as a function of \( n \), and let \( p|_{\text{J}}(n) \) denote a similar price function satisfying equation (12). Equilibrium is characterized by the intersection of these two price functions making \( p|_{\text{H}}(n) = p|_{\text{J}}(n) \equiv p^* \). The value of \( n \) solving this equation is the equilibrium amount \( n^* \). Notice that \( \lim_{n \to +\infty} p|_{\text{H}}(n) = 0 \) and \( \lim_{n \to 0} p|_{\text{H}}(n) = I/[(1 - \phi_1)n_B] \). With respect to \( p|_{\text{J}}(n) \), we get

\[
p|_{\text{J}}(n) \leq 0 \iff n \geq \frac{(1 - \theta)V_G}{c - \frac{\theta y k_B D}{m + n_B}} - m;
\]

35
and

\[ \lim_{n \to 0} p|_J(n) = \frac{c - \frac{(1-\theta)V_G}{m} - \frac{\theta y k_B D}{m + n_B}}{\frac{\theta y n k_B}{m + n_B} - [1 - (1 - y)\theta]} . \]

We now analyse the possible intersections of the functions \( p|_H \) and \( p|_J \). First, if \( p|_J(0) \geq p|_H(0) \), equilibrium is guaranteed since \( \lim_{n \to +\infty} p|_H(n) > \lim_{n \to +\infty} p|_J(n) \). Also notice that \( p|_J(0) \geq p|_H(0) \) is equivalent to

\[ c < \tau \equiv \frac{(1 - \theta)V_G}{m} + \frac{\theta y k_B D}{m + n_B} + \frac{I}{(1 - \phi_1)n_B} \left\{ \frac{\theta y n k_B}{m + n_B} - [1 - (1 - y)\theta] \right\} . \]

The existence of equilibrium is thus guaranteed if \( c < \tau \). Second, if \( p|_J(0) < p|_H(0) \Rightarrow c > \tau \), there may be no equilibrium. Let \( n_L \) be defined such that \( p|_H(n_L) = p_L \). A sufficient condition for intersection to exist is to take \( p|_J(n_L) > p_L \). This condition is algebraically equivalent to

\[ c < \tilde{\tau} \equiv \frac{(1 - \theta)V_G}{m} + \frac{\theta y k_B D}{m + n_B} + \frac{I}{\phi_1 p^*} \left\{ \frac{\theta y n k_B}{m + n_B} - [1 - (1 - y)\theta] \right\} . \]

We thus ensure equilibrium for any \( c < \tilde{\tau} \equiv \max(\tilde{\tau}, \tau) \).

The statement of the Proposition is completed as follows. If the firm is good \textit{ex-post}, the amount of funds collected is

\[ pn = I \frac{n}{\phi_1 n + (1 - \phi_1)n_B} . \]

Since \( \phi_1 \in (0, 1) \Rightarrow pn > I \). If the firm is bad \textit{ex-post}, the raised amount of funds in the going public operation is

\[ pn_B = I \frac{n_B}{\phi_1 n + (1 - \phi_1)n_B} . \]

Since \( \phi_1 \in (0, 1) \Rightarrow pn_B < I \).

For the above solution to be an equilibrium we must prove that any attempt to deviate from that solution will lead to a worse result.

For the equilibrium to exist, the \textit{ex-ante} bad and \textit{ex-ante} good firms cannot have gains in deviating from the pooling equilibrium, offering a number of shares and a price different from \( n^* \) and \( p^* \). Let us see what occurs if these firms deviate from the pooling equilibrium and are identified as \textit{ex-ante} bad firms.

If the \textit{ex-ante} bad firm deviate from the pooling equilibrium, the probability assessed by an uninformed investor that a firm offering \( n_{sb}^* \) shares (different from \( n^* \)) at a price \( p_{sb}^* \) (different from \( p^* \)) is an \textit{ex-post} bad firm is

\[ \theta_{sb} = (1 - \phi_2) . \]  

(27)
The *ex-ante* bad firm can establish a price different from the one given by equation (14). In fact, this firm will establish a price that enables it to obtain, in expected terms, the amount $I$ to invest

$$p = \frac{I}{\phi_2 n + (1 - \phi_2)n_B}.$$

(28)

The informed investor continues to have an expected gain equal to zero

$$(1 - \theta_{sb}) \frac{V_G}{m + n_{sb}} + \theta_{sb} y \frac{k_B (D + n_Bp)}{m + n_B} - [1 - (1 - y)\theta_{sb}] p - c = 0.$$  

(29)

Notice that the production of information continues to exist because the uncertainty about the firm’s type remains. The bad firm continues to sell $n_B = N_B y$ shares. In conclusion, the number of shares offered $n_{sb}^*$ and the price $p_{sb}^*$ are determined by equations (11, 27, 28) and (29). This system of equation has two differences in relation to the system that determines $n^*$ and $p^*$: $\theta_{sb} > \theta$ and the form of defining the price by the bad firm is different.

Next, we analyze the impact of each difference separately. First, $p_{sb}^*$ solves

$$F_{sb}(p_{sb}) = \frac{(1 - \theta_{sb}) \phi_2 V_G p_{sb}}{p_{sb} (\phi_2 m - (1 - \phi_2)n_B) + I} + \frac{\theta_{sb} y k_B (D + p_{sb}n_B)}{m + n_B} - [1 - (1 - y)\theta_{sb}] p_{sb} - c = 0.$$  

Notice that

$$\frac{\partial}{\partial \phi} \left[ \frac{(1 - \theta) \phi V_G p}{p (\phi m - (1 - \phi)n_B) + I} \right]_{\phi = \phi_1} > \frac{(1 - \theta) V_G p}{p (\phi m - (1 - \phi)n_B) + I} > 0.$$  

Since $\phi_1 > \phi_2$, we have

$$\frac{(1 - \theta) \phi_1 V_G}{p (\phi_1 m - (1 - \phi_1)n_B) + I} > \frac{(1 - \theta) \phi_2 V_G}{p (\phi_2 m - (1 - \phi_2)n_B) + I}.$$  

Second, we have $\theta_{sb} > \theta$. Notice that we have

$$\frac{\partial F_{sb}(p_{sb})}{\partial \theta} = -\frac{1}{m + n_{sb}} V_G + \frac{\theta k_B (D + p_{sb}n_B)}{m + n_B} + (1 - y)p_{sb}.$$  

The uninformed investor has a negative expected gain, meaning that

$$(1 - \theta) \frac{V_G}{m + n_{sb}} + \frac{\theta k_B (D + p_{sb}n_B)}{m + n_B} \leq p_{sb}.$$  

\textsuperscript{18}Without taking into account the impact of $\phi$ on $\theta$.  

37
Since \( \frac{V_G}{m+n_b} > \frac{k_B(D+p_{sh}n_B)}{m+n_B} \), we have \( \frac{V_G}{m+n_b} > p_{sb} > \frac{k_B(D+p_{sh}n_B)}{m+n_B} \). This means that \( \partial F_{sb}(p_{sb})/\partial \theta < 0 \).

In conclusion, with \( \phi_1 > \phi_2 \) and \( \theta_{sb} > \theta \) we have \( F_{sb}(p^*) < 0 \), where \( p^* \) solves \( F(p^*) = 0 \). As above, define \( k_B < \frac{k^2_B}{l_B} \) such that \( \partial F_{sb}(p_{sb})/\partial p_{sb} < 0 \) for all \( 0 < p_{sb} < I/[(1 - \phi_2)n_B] \). Since \( F_{sb}(p^*) < 0 \) and \( \partial F_{sb}(p_{sb})/\partial p_{sb} < 0 \), we have \( p^*_{sb} < p^* \). This also implies, using equations (14) and (28) that \( n^*_{sb} > n^* \).

In conclusion, if the ex-ante bad firm separates, issues more shares at a lower price. This leads to an expected gain for the firm in the separating equilibrium lower than the expected gain in the pooling equilibrium.

To study the separating strategy of the good firm, define, as above, \( k_B < \frac{k^2_B}{l_B} \) such that \( \partial F_{sg}(p_{sg})/\partial p_{sg} < 0 \) for all \( 0 < p_{sg} < I/[(1 - \phi_2)n_B] \), where \( p_{sg} \) is the price offered by the firm under this strategy. After that we can prove, as above, that \( p^*_{sg} < p^* \) and \( n^*_{sg} > n^* \), implying that the ex-ante good firm losses in deviating from the pooling equilibrium.

Finally, defining \( \tilde{k}_B \equiv \min(\frac{1}{k^2_B}, \frac{k^2_B}{l_B}, \frac{k^2_B}{k^2_B}) \) the result of the proposition follows.

### 8.4 Proof of Corollary 4

First, let us see the impact of \( c \) in the share price. We have

\[
\frac{\partial p^*}{\partial c} = -\frac{\partial F(p)}{\partial c} \frac{\partial F(p)}{\partial p} < 0,
\]

because \( \partial F(p)/\partial c = -1 \) and \( \partial F(p)/\partial p < 0 \). Let us determine the impact of \( I \) on \( p^* \). We obtain first

\[
\frac{\partial F(p)}{\partial I} = \frac{(1 - \theta)\phi_1 p k_G \{p [\phi_1 m - (1 - \phi_1) n_B] - D\}}{\{p [\phi_1 m - (1 - \phi_1) n_B] + I\}^2}.
\]

On one hand, \( n_B > \frac{1 - \phi_2}{\phi_1} m \Rightarrow \partial F(p)/\partial I < 0 \) and

\[
\frac{\partial p^*}{\partial I} = -\frac{\partial F(p)}{\partial I} \frac{\partial F(p)}{\partial p} < 0. \tag{30}
\]

On the other hand, if \( n_B < \frac{1 - \phi_2}{\phi_1} m \) the signal of \( \partial F(p)/\partial I \) is ambiguous. In this case, with either \( D = 0 \) or \( D < p^* [\phi_1 m - (1 - \phi_1) n_B] \) we have \( \partial F(p)/\partial I > 0 \) and

\[
\frac{\partial p^*}{\partial I} > 0. \tag{31}
\]
However, with $D > p^*[\phi_1 m - (1 - \phi_1)n_B]$ we get $\frac{\partial F(p)}{\partial I} < 0$ and

$$\frac{\partial p^*}{\partial I} < 0.$$  \hfill (32)

Next, we study the impact of $D$ on $p^*$. We have

$$\frac{\partial F(p)}{\partial D} = \frac{(1 - \theta)\phi_1 k_gp}{p(\phi_1 m - (1 - \phi_1)n_B) + I} + \frac{\theta yk_B}{m + n_B} > 0$$

and therefore $\frac{\partial p^*}{\partial D} > 0$.

Finally, we have

$$\frac{\partial F(p)}{\partial k_G} = \frac{(1 - \theta)\phi_1(D + I)p}{p(\phi_1 m - (1 - \phi_1)n_B) + I} > 0$$

and $\frac{\partial p^*}{\partial k_G} > 0$.

### 8.5 Proof of Proposition 5

The *ex-ante* good firm is indifferent between the VC and the public market if its expected gain is the same in the two financing options, *i.e.*, \(^{19}\)

$$Q_G = [\text{Good Firm EG VC}] - [\text{Good Firm EG P}] = 0$$

$$= \phi_1(1 - s_g^*)k_G (D + I) + (1 - \phi_1)k_B (D + I) \left[ y(1 - s_g^*) + (1 - y)(1 - s_g^*) \right] + \mu(1 - \phi_1)(1 - y)C_R - \left[ \frac{m}{m + n^*}k_G (D + I) + \frac{m}{m + n_B}k_B(D + p^*n_B) + \mu(1 - \phi_1)C_R \right]$$

Let us define $\rho_G(c)$ as the function that solves $Q_G = 0$. We have

$$\frac{\partial \rho_G}{\partial c} = -\frac{\partial Q_G}{\partial c}$$

On one hand, after some calculations, we get

$$\frac{\partial Q_G}{\partial \rho} = [\phi_1 k_G (D + I) + (1 - \phi_1)yk_B (D + I)] \left(-\frac{\partial s_g^*}{\partial \rho}\right)$$

$$+ (1 - \phi_1)(1 - y)k_B (D + I) \left(-\frac{\partial s_g^*}{\partial \rho}\right).$$

\(^{19}\) Good Firm EG VC\(^n\) means good firm’s expected gain with VC financing and “Good Firm EG P\(^n\)” means good firm’s expected gain with public financing.
Since $\partial s^*_g/\partial \rho > 0$ and $\partial s^*_b/\partial \rho > 0$, we have $\partial Q_G/\partial \rho < 0$. On the other hand, we have first

$$
\frac{\partial \text{[Good Firm EG VC]}}{\partial c} = [\phi_1 k_G (D + I) + (1 - \phi_1) y k_B (D + I)] (\frac{\partial s^*_g}{\partial c}) + (1 - \phi_1) (1 - y) k_B (D + I) (\frac{\partial s^*_b}{\partial c}).
$$

Since $\partial s^*_g/\partial c > 0$ and $\partial s^*_b/\partial c > 0$, we have $\partial \text{[Good Firm EG VC]}/\partial c < 0$. Second, we get

$$
\frac{\partial \text{[Good Firm EG P]}}{\partial c} = \left[ \phi_1 m k_G (D + I) \left( \frac{\partial n^*}{\partial c} \right)^2 + \frac{(1 - \phi_1) m n_B k_B \partial p^*}{m + n_B} \frac{\partial c}{\partial c} \right].
$$

Given that $\partial n^*/\partial c > 0$ and that $\partial p^*/\partial c < 0$, we have $\partial \text{[Good Firm EG P]}/\partial c < 0$. This implies that the signal of $\partial Q_G/\partial c$ is ambiguous. Hence, we assume that

$$
\frac{\partial Q_G}{\partial c} > 0. \quad (33)
$$

This implies that we get $\partial \rho_G/\partial c > 0$. We have to impose the same conditions for the bad firm. For this firm, the indifference between the two forms of financing is given by

$$
Q_B \equiv \phi_2 (1 - s^*_g) k_G (D + I) + (1 - \phi_2) [y (1 - s^*_g) k_B (D + I) + (1 - y) (1 - s^*_b) k_B (D + I)] + \\
\mu (1 - \phi_1) (1 - y) C_R - \\
\phi_2 \frac{m}{m + n^*} k_G (D + I) + (1 - \phi_2) \frac{m}{m + n_B} k_B (D + p^* n_B) + \mu (1 - \phi_2) C_R = 0.
$$

Let us define $\rho_B(c)$ as the function that solves $Q_B = 0$. As for the ex-ante good firm, we assume that

$$
\frac{\partial Q_B}{\partial c} > 0. \quad (34)
$$

This implies that we get $\partial \rho_B/\partial c > 0$. Let $\rho_1$ be the concrete VC’s coefficient of risk-aversion. The parameter $c_G$ solves $Q_G(c_G, \rho_1) = 0$. Similarly, define
$c_B$ for the \textit{ex-ante} bad firm, \textit{i.e.}, $Q_B(c_B, \rho_1) = 0$. Rewrite $Q_G$ as

$$Q_G = \phi_1 \left[ (1 - s_g^*) k_G I - y (1 - s_g^*) k_B I - (1 - y) (1 - s_B^*) k_B I - \mu (1 - y) C_R \right] +$$

$$y (1 - s_g^*) k_B I + (1 - y) (1 - s_B^*) k_B I + \mu (1 - y) C_R -$$

$$\phi_1 \left[ \frac{m}{m + n^*} k_G (D + I) - \frac{m}{m + n_B^*} k_B (D + p^* n_B) - \mu C_R \right] -$$

$$\frac{m}{m + n_B} k_B (D + p^* n_B) + \mu C_R$$

$$= \phi_1 \Psi_1 + \Psi_2 - \phi_1 \Psi_3 - \Psi_4.$$ 

Doing the same for $Q_B$, we get $Q_B = \phi_2 \Psi_1 + \Psi_2 - \phi_2 \Psi_3 - \Psi_4$. Therefore, we obtain

$$Q_G - Q_B = [\Psi_1 - \Psi_3] [\phi_1 - \phi_2].$$

With respect to the difference $[\Psi_1 - \Psi_3]$ we may have three situations. In Situation 1 we have

$$[\Psi_1 - \Psi_3]_{c_G, \rho_1} > 0,$$

implying $Q_G(c_G, \rho_1) - Q_B(c_G, \rho_1) > 0 \iff Q_B(c_G, \rho_1) < 0$. This implies that $c_B > c_G$, because $\partial Q_B(c, \rho) / \partial c > 0$. In Situation 2 we have

$$[\Psi_1 - \Psi_3]_{c_G, \rho_1} < 0,$$

implying $Q_B(c_G, \rho_1) > 0$ and $c_B < c_G$. In Situation 3 we have $[\Psi_1 - \Psi_3]_{c_G, \rho_1} = 0$, implying $Q_B(c_G, \rho_1) = 0$ and $c_B = c_G$. Notice that

$$\frac{\partial [\text{Good Firm EG VC}]}{\partial \phi_1} = \Psi_1 + \phi_1 \frac{\partial \Psi_1}{\partial \phi_1} + \frac{\partial \Psi_2}{\partial \phi_1}.$$

The same occurs for the bad firm. This means that $\Psi_1$ measures the direct impact of a change in $\phi$ in the firm’s expected gain when exists VC financing. In the same way

$$\frac{\partial [\text{Good Firm EG P}]}{\partial \phi_1} = \Psi_3 + \phi_1 \frac{\partial \Psi_3}{\partial \phi_1} + \frac{\partial \Psi_4}{\partial \phi_1}.$$

The same happens for the bad firm. Therefore $\Psi_3$ measures the direct impact of a change in $\phi$ in the firm’s expected gain when exists public financing.

In Situation 1, with $c < c_G$ both types of firms choose the public financing (see Figure (3)). With $c_G < c < c_B$ the \textit{ex-ante} good firm chooses the VC financing and the \textit{ex-ante} bad firm chooses public financing (without taking into account the other’s firm decision). Finally, with $c > c_B$ both types of firms choose the VC financing.
If the *ex-ante* bad firm chooses the public financing with \( c_G < c < c_B \), it will be identified as an *ex-ante* bad firm. Thus, with \( c_G < c < c_B \) the actual choice for the *ex-ante* bad firm is between the VC and the public financing with separation from the *ex-ante* good firm.

In \( c = c_B \), the *ex-ante* bad firm is indifferent between the VC and the public financing with pooling. We know that the public financing in a pooling situation is better than in a separating situation. Thus, in \( c = c_B \) the firm prefers the VC financing to the public financing in a separating situation.

Let us define \( Q_B^{S_1} \) as the difference between the *ex-ante* bad firm gain with VC financing and with public financing in a separating situation, such that

\[
Q_B^{S_1} = \phi_2 (1 - s^*_y) k_G (D + I) + (1 - \phi_2) y (1 - s^*_y) k_B (D + I) + (1 - y) (1 - s^*_b) k_B (D + I) + \\
\mu (1 - \phi_2) (1 - y) C_R - \\
\left[ \phi_2 \frac{m}{m + n^*_s} k_G (D + I) + (1 - \phi_2) \frac{m}{m + n_B} k_B (D + p^*_s n_B) + \mu (1 - \phi_2) C_R \right].
\]

Like above, we assume

\[
\frac{\partial Q_B^{S_1}}{\partial c} > 0. \tag{37}
\]

With \( c = c_B \), \( Q_B^{S_1} > 0 \). In order to have \( Q_B^{S_1} = 0 \), \( c \) must decrease. The parameter \( c_B^{S_1} \) solves \( Q_B^{S_1} = 0 \) and \( c_B^{S_1} < c_B \). However, we do not know if \( c_B^{S_1} > c_G \). If \( c_B^{S_1} > c_G \), then with \( c_G < c < c_B^{S_1} \), the *ex-ante* bad firm chooses the public financing in a separating situation, and with \( c_B^{S_1} < c < c_B \) chooses

<table>
<thead>
<tr>
<th>Good Firm</th>
<th>Public pooling</th>
<th>VC pooling</th>
<th>VC pooling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bad Firm</td>
<td>Public pooling</td>
<td>Public pooling</td>
<td>VC pooling</td>
</tr>
<tr>
<td>Bad Firm</td>
<td>Public pooling</td>
<td>Public Sepa. pool</td>
<td>VC pooling</td>
</tr>
</tbody>
</table>

\[
\text{If } c_B^{S_1} < c_G
\]

\[
\text{If } c_B^{S_1} > c_G
\]

Figure 3: Global equilibrium in Situation 1.
the VC financing. If $c_B^{S1} < c_G$, then with $c_G < c < c_B$, both firms choose the VC financing.

In Situation 2, with $c < c_B$ both types of firms choose the public financing. With $c_B < c < c_G$ the ex-ante good firm chooses the public financing and the ex-ante bad firm chooses VC financing (without taking into account the other firm’s decision). Finally, with $c > c_G$ both types of firms choose the VC financing (see Figure (4)).

If the ex-ante bad firm chooses the VC financing with $c_B < c < c_G$, the bad firm separates from the ex-ante good firm. In $c = c_B$ the bad firm prefers the public financing to the VC financing in a separating situation.

Let us define $Q_B^{S2}$ as the difference between the gain of the ex-ante bad firm with VC financing in a separating situation and with public financing, i.e.,

$$Q_B^{S2} = \phi_2 (1 - s_{2B}^*) k_G (D + I) + (1 - \phi_2) \left[ y(1 - s_{2B}^*) k (D + I) + (1 - y)(1 - s_{2B}^*) k_B (D + I) \right] + \mu (1 - \phi_2)(1 - y) C_R - \mu (1 - \phi_2) C_R - \left[ \phi_2 \frac{m}{m + n_B^*} k_G (D + I) + (1 - \phi_2) \frac{m}{m + n_B} k_B (D + p_B n_B) + \mu (1 - \phi_2) C_R \right].$$

As usual, we assume

$$\frac{\partial Q_B^{S2}}{\partial c} > 0.$$  
(38)

With $c = c_B$, $Q_B^{S2} < 0$. The parameter $c_B^{S2}$ solves $Q_B^{S2} = 0$ and $c_B^{S2} > c_B$. However, we do not know if $c_B^{S2} < c_G$. If $c_B^{S2} < c_G$, then with $c_B < c < c_B^{S2}$, the ex-ante bad firm chooses the public financing, and with $c_B^{S2} < c < c_G$
chooses the VC financing in a separating situation. If \( c_c^{S2} > c_g \), then with 
\( c_B < c < c_c \) both firms choose the public financing.

In this paragraph we analyze intuitively the existence of a separating behavior by the bad firm in Situation 2. The bad firm chooses to separate from the good firm between \( c_B < c < c_c^{S2} \) if its expected gain when chooses the VC in a separating situation \( (G_{VCSep}^{B}) \) is higher than its expected gain when chooses the public market \( (G_{Pub}^{B}) \), i.e., \( G_{VCSep}^{B} > G_{Pub}^{B} \). The last expression is equivalent to

\[
G_{VCSep}^{B} - G_{VCPool}^{B} + G_{VCPool}^{B} - G_{Pub}^{B} > 0, \tag{39}
\]

where \( G_{VCPool}^{B} \) is the bad firm’s expected gain when chooses the VC financing in a pooling situation. We know that \( G_{VCPool}^{B} - G_{Pub}^{B} > 0 \), with \( c_g > c > c_B \), and that \( G_{VCSep}^{B} < G_{VCPool}^{B} \), as proved in Proposition 1. Thus, the condition (39) is only verified if \( G_{VCPool}^{B} - G_{VCSep}^{B} \) is not too large. This only occurs if \( s_{B}^{*} - s_{B}^{*} \) is not too large. The last difference is not too large if \( V_g - V_{gB} \) is not too large. We know that \( V_g - V_{gB} \) depends on the difference between \( \Pr(q = G | e = g) \) and \( \Pr(q = G | e = g, Bad) \). Finally, from inequality (19), we get that \( G_{VCPool}^{B} - G_{VCSep}^{B} \) is not too large when \( \phi_1 - \phi_2 \) is not too large also.

### 8.6 Proof of Corollary 6

We have \( \frac{\partial Q_g}{\partial \mu} = -(1 - \phi_1)C_{RY} < 0 \) and \( \frac{\partial Q_g}{\partial \mu} = -(1 - \phi_2)C_{RY} < 0 \). Notice that \( \mu \) does not has impact on the decision variables, \( s_{g}^{*}, s_{B}^{*}, n^{*}, p^{*} \). We also have

\[
\frac{\partial \rho_G}{\partial \mu} = -\frac{\partial Q_g}{\partial \mu} = \frac{\partial Q_g}{\partial \mu} < 0.
\]

The variable \( c_g \) solves \( M = \rho_G(c) - \rho_1 = 0 \). Therefore, we have

\[
\frac{\partial c_g}{\partial \mu} = -\frac{\partial M}{\partial \mu} = -\frac{\partial \rho_G}{\partial \mu} > 0.
\]

The variable \( c_B \) solves \( \rho_B(c) - \rho_1 = 0 \). Applying the same reasoning, we conclude that an increase in \( \mu \), increases also \( c_B \).

In Situation 1, with \( c_B > c_g \), a higher \( \mu \) means that firms go public for higher \( c \) (see Figure 5\(^{20} \)). This affirmation remains true if the bad firm separates from the good firm between \( c_g \) and \( c_B \), because a higher \( \mu \) also implies a higher \( c_c^{S1} \).

\(^{20}\)In this figure and in the next figures, we only represent the choice that each firm do without taking into account the other firm’s decision.
Figure 5: The impact of $\mu$ in the global equilibrium in Situation 1.

In Situation 2, when $c_B < c_G$, an increase in $\mu$ also means that the firms go public for a higher $c$ (see Figure (6)). This affirmation remains true if the bad firm separates from the good firm (between $c_G$ and $c_B$), since an increase in $\mu$ increases $c_{G2}^2$.

When $\mu$ increase, we cannot guarantee that $[\Psi_1 - \Psi_3]_{CG,\rho_1}$ maintains its previous signal. Therefore, let us study what happens if $[\Psi_1 - \Psi_3]_{CG,\rho_1}$ changes signal when $\mu$ increases. In Situation 1, the firms continues to go public for higher $c$ when $\mu$ increases (see Figure 7). The same happens in Situation 2.

In conclusion, when $\mu$ increases, $c_B$ and $c_G$ increase and therefore the going public space expand.

8.7 Proof of Corollary 7

In first place, an increase in $N$ increases $n_B$. It then follows that the increase in $n_B$ has an impact on $p^*$ and $n^*$. In fact, we have

$$\frac{\partial F(p^*)}{\partial n_B} = \frac{(1 - \theta)\phi_1(1 - \phi_1)V_{GP}s^2}{[p^*(\phi_1 m - (1 - \phi_1)n_B) + I]^2} + \frac{\theta y k_B (p^* m - D)}{[m + n_B]^2}.$$

From inequality (13), we know that $p^* > \frac{k_B(D + p^* n_B)}{m + n_B}$, which implies $p^* m - D > 0$ and $\partial F(p)/\partial n_B > 0$. This leads to

$$\frac{\partial p^*}{\partial n_B} = \frac{\partial F(p^*)}{\partial n_B} > 0$$
Figure 6: The impact of $\mu$ in the global equilibrium in Situation 2.

Figure 7: The impact of $\mu$ in the global equilibrium when $[\Psi_1 - \Psi_3]_{cG,\rho_1}$ changes signal with the increase in $\mu$. 
and \( \partial n^*/\partial n_B < 0 \). Additionally, we have
\[
\frac{\partial Q_G}{\partial n_B} = -\phi_1 mk_G (D + I) \frac{-\frac{\partial n^*}{\partial n_B}}{(m + n^*)^2} \frac{(1 - \phi_1)mk_B \left[ \frac{\partial \rho_{s_B}}{\partial n_B} n_B (m + n_B) + p^* m - D \right]}{(m + n_B)^2} < 0.
\]
This implies that
\[
\frac{\partial \rho_G}{\partial n_B} = -\frac{\partial Q_G}{\partial \rho} < 0.
\]
We thus have
\[
\frac{\partial c_G}{\partial n_B} = -\frac{\partial M}{\partial c_G} = -\frac{\partial \rho_G}{\partial \rho} > 0.
\]
Similarly, \( c_B \) also increases. As we saw in the Proof of Corollary 6, an increase in \( c_B \) and \( c_G \) is a sufficient condition for the going public zone to increase.

### 8.8 Proof of Remark 8

First, let us analyze the impact of \( I \) in the decision of reverting the LBO. On one hand, we have
\[
\frac{\partial (\text{Good Firm EG P})}{\partial I} = \phi_1 mk_G \frac{m + n^* - (D + I) \frac{\partial n^*}{\partial I}}{(m + n^*)^2} + \frac{(1 - \phi_1)mn_B k_B \frac{\partial p^*}{\partial I}}{m + n_B}.
\]
Since we do not know the signal of \( \frac{\partial p^*}{\partial I} \) and \( \frac{\partial n^*}{\partial I} \), the last expression has an ambiguous signal. However, when \( \frac{\partial p^*}{\partial I} > 0 \) and \( \frac{\partial n^*}{\partial I} < 0 \), we have
\[
\frac{\partial (\text{Good Firm EG P})}{\partial I} > 0.
\]
On the other hand,
\[
\frac{\partial (\text{Good Firm EG VC})}{\partial I} = \phi_1 (1 - s^*_g)k_G + \phi_1 (-\frac{\partial s^*_g}{\partial I})k_G (D + I) + \frac{y(1 - s^*_g)k_B + y(-\frac{\partial s^*_g}{\partial I})k_B (D + I)}{(1 - \phi_1)} + \frac{(1 - y)(1 - s^*_b)k_B + (1 - y)(-\frac{\partial s^*_b}{\partial I})k_B (D + I)}{(1 - \phi_1)}.
\]
Since we do not know the signal of \( \frac{\partial s^*_g}{\partial I} \) and \( \frac{\partial s^*_b}{\partial I} \), the last expression has an ambiguous signal. However, if \( \frac{\partial s^*_g}{\partial I} < 0 \) and \( \frac{\partial s^*_b}{\partial I} < 0 \), the firm’s
expected gain increase with the increase in $I$. In conclusion, generically the
signal of $\partial Q_G / \partial I$ is ambiguous. The same occurs for the signal of $\partial Q_B / \partial I$.

Second, we analyze the impact of $D$ in the going public decision. We obtain
\[
\frac{\partial}{\partial D} \left( \text{Good Firm EG P} \right) = \phi_1 m k_G \frac{m + n^* - (D + I) \frac{\partial n^*}{\partial D}}{(m + n^*)^2} + \frac{(1 - \phi_1) m k_B}{m + n_B} \left(1 + n_B \frac{\partial p^*}{\partial D} \right) > 0.
\]
and
\[
\frac{\partial}{\partial D} \left( \text{Good Firm EG VC} \right) = \phi_1 (1 - s^*_g) k_G + \phi_1 \left( - \frac{\partial s^*_g}{\partial D} \right) k_G (D + I) + (1 - \phi_1) \left[ y(1 - s^*_g) k_B + y \left( - \frac{\partial s^*_g}{\partial D} \right) k_B (D + I) \right] + (1 - \phi_1) \left[ (1 - y)(1 - s^*_b) k_B + (1 - y) \left( - \frac{\partial s^*_b}{\partial D} \right) k_B (D + I) \right] > 0
\]
Therefore, the signal of $\partial Q_G / \partial D$ is ambiguous. The same occurs for the
signal of $\partial Q_B / \partial D$.

Finally, let us study the impact of $k_G$ on the going public decision. We get
\[
\frac{\partial}{\partial k_G} \left( \text{Good Firm EG P} \right) = \phi_1 m \frac{(D + I) (m + n^*) - k_G (D + I) \frac{\partial n^*}{\partial k_G}}{(m + n^*)^2} + \frac{(1 - \phi_1) m k_B n_B}{m + n_B} \frac{\partial p^*}{\partial k_G} > 0
\]
and
\[
\frac{\partial}{\partial k_G} \left( \text{Good Firm EG VC} \right) = \phi_1 (1 - s^*_g) (D + I) + \phi_1 \left( - \frac{\partial s^*_g}{\partial k_G} \right) k_G (D + I) + (1 - \phi_1) \left[ y(1 - s^*_g) + y \left( - \frac{\partial s^*_g}{\partial k_G} \right) k_B + (1 - y)(1 - s^*_b) + (1 - y) \left( - \frac{\partial s^*_b}{\partial k_G} \right) k_B \right] > 0.
\]
This implies that the signal of $\partial Q_G / \partial k_G$ is ambiguous. The same occurs for
the signal of $\partial Q_B / \partial k_G$. 

48
References


