The Output Effects of (Non-Separable) Government Consumption at the Zero Lower Bound

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Abstract

We investigate the reaction of output to government spending shocks at the zero lower bound (ZLB) on the nominal interest rate when government and private consumption are non-separable in preferences. In particular, substitutability between private and government consumption significantly reduces the otherwise large output multipliers obtained at the ZLB. Additionally, the coupling of substitutability with a debt-stabilizing fiscal rule can generate negative output multipliers on impact.

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1 Introduction

Since the end of 2008, nominal interest rates have moved to the ZLB across major developed economies. Christiano et al. (2011), henceforth CER (2011), show that, within a New

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Keynesian (NK) model, fiscal policy can be particularly effective in boosting output when the nominal interest rate is at the ZLB. To see why, suppose, as in Eggertsson and Woodford (2003), that the occurrence of a given shock increases desired savings. Because of price stickiness and the ZLB, the fall in the real interest rate might be insufficient in re-establishing the equilibrium. In this situation, desired savings must decrease, which only occurs with a potentially sharp reduction in output. At this point, an increase in government spending produces, all else equal, an upward pressure on expected future inflation which, in turn, translates into a lower real interest rate. This mitigates the fall in output needed to restore the equilibrium and adds to the standard upward shift of labor supply generated by the expansion in government spending. Thus, the output multipliers can be significantly bigger than the ones obtained when the nominal interest rate is far above the ZLB.

The dynamic above assumes that government consumption is either pure waste or enters non-separably in the household’s utility function. However, this assumption has been questioned by several works. Among others, Aschauer (1985), Ahmed (1986) and Ercolani and Valle e Azevedo (2012) find substitutability between private and government consumption, as in the model suggested by Barro (1981). Importantly, this substitutability tames the positive reaction of output to government consumption shocks, *ceteris paribus*. That is, an increase in government consumption makes private consumption less enjoyable, or, the marginal utility of private consumption decreases. This leads agents to partially substitute private consumption with newly available government consumption. For example, rises in public health care spending can reduce the need for private health services, and boosts to public education services can reduce the need for private schools and tutors. As a result, aggregate demand and labor supply increase relatively less than in a world with separable government consumption.

In this paper, we challenge the finding that government spending multipliers are large when the ZLB binds by introducing substitutability between private and government consumption. We document that this substitutability significantly affects the size of the output
multipliers at the ZLB. In a Ricardian world, we find that the output multipliers generated by the model with substitutability are roughly two thirds of the ones associated to the model with separabilities. Further, we show that the coupling of substitutability with a debt-stabilizing fiscal rule, as in Leeper et al. (2010), Uhlig (2010) or Traum and Yang (2011), can generate negative output multipliers on impact.

2 Model

We use an otherwise standard NK set-up similar to a vast class of models, e.g., Schmitt-Grohé and Uribe (2006), henceforth SGU (2006), and Smets and Wouters (2007). We deviate from these models in that we allow government consumption to affect the household’s marginal utility of consumption. We maintain various empirically plausible elements of these previous models which have proved useful in providing a good fit to the data. In what follows, we simplify the exposition of the micro-foundations of the model, as they are now standard.

2.1 Households

The economy is populated by a large representative household composed of a continuum of members indexed by $h \in [0, 1]$. The household derives utility from effective consumption, $\tilde{C}_t$, and disutility from working $L_t$, where $L_t = \left[ \int_0^1 L_t(h) \frac{\varepsilon_w - 1}{\varepsilon_w} dh \right]^{\frac{1}{\varepsilon_w - 1}}$, $L_t(h)$ is the quantity of labor of type $h$ supplied and $\varepsilon_w$ is the elasticity of substitution across varieties. $L_t$ is supplied by labor packers to intermediate goods firms in a competitive market at cost $W_t = \left[ \int_0^1 W_t(h)^{1-\varepsilon_w} dh \right]^{\frac{1}{1-\varepsilon_w}}$, where $W_t(h)$ is the price of each labor variety. Effective consumption is assumed to be an Armington aggregator of private consumption, $C_t$, and government consumption, $G_t$:

$$\tilde{C}_t = \left[ \phi \left( C_t \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1 - \phi) G_t^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

(1)

where $\phi \in [0, 1]$, and $\nu \in (0; \infty)$ is the elasticity of substitution between $C_t$ and $G_t$. Conditional on $\phi < 1$, large values of $\nu$ make $C_t$ and $G_t$ substitutes. If $\phi = 1$ then $\tilde{C}_t = C_t$ and
the standard hypothesis of separability emerges. In turn, \( C_t \) is a bundle of goods \( C_t(j) \), with \( j \in [0, 1] \), assembled by a final goods firm operating in competitive markets and given by \( C_t = \left[ \int_0^1 C_t(j) \frac{\varepsilon}{\varepsilon + 1} dj \right]^\frac{1}{\varepsilon + 1} \), where \( \varepsilon \) is the elasticity of substitution across varieties of goods. This bundle costs \( P_t = \left[ \int_0^1 P_t(j) (1-\varepsilon) dj \right]^\frac{\varepsilon}{\varepsilon + 1} \), where \( P_t(j) \) is the price of each variety. The lifetime expected utility of the representative household is given by:

\[
E \sum_{t=0}^{\infty} (e^{\lambda t} \beta)^t \left[ \frac{(\tilde{C}_t - \theta \tilde{C}^A_{t-1})^{1-\sigma_c}}{1-\sigma_c} - \chi \frac{L^1_{t+1} + \sigma_L}{1+\sigma_L} \right],
\]

where \( \sigma_c \) denotes the degree of relative risk aversion, \( \sigma_L \) is the inverse of the Frisch elasticity of labor supply, \( \theta \in (0; 1) \) measures the degree of habit formation in (aggregate) effective consumption \( \tilde{C}^A_t \), \( \beta \in (0, 1) \) is the subjective discount factor, and \( \chi \) is a preference parameter. \( \lambda_t \) represents a discount factor shock, assumed to follow a first-order autoregressive process with an i.i.d. error term: \( \lambda_t = \rho \lambda_{t-1} + \eta_t^\lambda \). As in CER (2011), this shock is crucial in bringing the economy to the ZLB. The representative household faces the following budget constraint in real terms:

\[
(1 + \tau^e)C_t + I_t + B_t = \frac{R_{t-1}}{\pi_t} B_{t-1} + (1 - \tau_t) \frac{1}{P_t} W_t L_t + (1 - \tau_t) \left[ r^K_t u_t - a(u_t) \right] \tilde{K}_t + D_t - T_t,
\]

where \( R_t \) is the gross nominal interest rate on governments bonds, \( B_t, \pi_t = \frac{R}{P_{t-1}} \) is the gross inflation rate, \( W_t L_t \) is labor income, \( \tilde{K}_t \) is the capital stock, \( D_t \) are the dividends paid by household-owned firms, and \( T_t \) are lump-sum taxes. \( \tau^e \) and \( \tau_t \) are tax rates on consumption and income, respectively. Following SGU (2006), the cost of using capital at intensity \( u_t \) is given by \( a(u_t) = \gamma_1 (u_t - 1) + \frac{\gamma_2}{2} (u_t - 1)^2 \). The effective capital, \( K_t = u_t \tilde{K}_t \), is rented to firms in a competitive market at cost \( r^K_t \). \( \tilde{K}_t \) evolves according to:

\[
\tilde{K}_t = (1 - \delta_k) \tilde{K}_{t-1} + I_t \left[ 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right],
\]
where $\delta_k$ is the depreciation rate and $\kappa$ governs the cost of changing the current level of investment $I_t$, relative to $I_{t-1}$.

The representative household maximizes her lifetime expected utility by choosing $C_t$, $B_t$, $K_t$, $I_t$, and $u_t$ subject to (3) and (4). Each of the members of the household supplies $L_t(h)$ units of labor while re-optimizing the (nominal) wage, $W_t(h)$, with probability $1 - \xi_w$ in each period $t$, where $\xi_w \in [0, 1]$. Members re-optimizing their wage maximize their expected utility in all states of nature in which they are unable to re-optimize in the future, subject to (3) and the demand for labor services, $L_{t+s}(h) = \left( \frac{W_t(h)}{W_{t+s}} \right)^{-\xi_w} L_{t+s}$, generated by the labor packers. Households who do not re-optimize at time $t$ index their wages according to the rule $W_t(h) = W_{t-1}(h)\pi_{t-1}^w$.

### 2.2 Firms

There is a continuum of household-owned monopolistic firms, indexed by $j \in [0, 1]$, each of which produces differentiated goods, $Y_t(j)$, using the following technology:

$$Y_t(j) = \max(K_t(j)^\alpha L(j)^{1-\alpha} - \Phi, 0),$$

(5)

where $Y_t(j)$ is the output of good $j$, $\alpha$ is the share of capital, and $\Phi$ represents a fixed cost of production. Capital, $K_t(j)$, and labor, $L(j)$, are obtained in competitive markets. At each period $t$, a share $1 - \xi_p$ of firms, where $\xi_p \in [0, 1]$, resets its price, $P_t(j)$. Firms resetting $P_t(j)$ in period $t$ maximize the expected present discounted value of dividends in the states of nature in which they are unable to re-optimize, i.e., they solve:

$$\max_{P_t(j)} E_t \left\{ \sum_{s=0}^{\infty} \xi_p^s \beta_{t,t+s} Y_{t+s}(j) [P_t(j) - MC_{t+s}] \right\},$$

(6)

subject to the demand $Y_{t+s}(j) = \left( \frac{P_t(j)}{P_{t+s}} \right)^{-\xi} Y_{t+s}$ generated by the final goods firm, where $Y_{t+s} = \left[ \int_0^1 Y_{t+s}(j) \frac{\xi-1}{\xi} dj \right]^\frac{1}{\xi}$. $\beta_{t,t+s}$ is the stochastic discount factor of the households and
$MC_t = \frac{(r_t^k)^{\alpha} W_t^{1-\alpha}}{\alpha^{(1-\alpha)}}$. Those firms which cannot re-optimize will instead index their prices according to the rule $P_t(j) = P_{t-1}(j)\pi_{t-1}^\rho$.

### 2.3 Fiscal and Monetary Policy

The government buys $G_t$ units of final goods each period. Its budget constraint is:

$$G_t + \frac{R_{t-1}}{\pi_t} B_{t-1} = B_t + \tau^c C_t + \tau_t \frac{W_t}{Y_t} L_t + \tau_t \left[ r_t^k u_t - a (u_t) \right] K_t + T_t,$$

(7)

We assume a first-order autoregressive process for $G_t$ with an i.i.d error term, i.e., $G_t = (1 - \rho G)G_{ss} + \rho G_{t-1} + \eta_t^G$, where $G_{ss}$ is the steady state level for $G$. Following Traum and Yang (2011), we assume the income tax rate follows:

$$\tau_t = (1 - \rho)\tau_{ss} + \rho \tau_{t-1} + (1 - \rho)\gamma \left( \frac{B_{t-1}}{Y_{t-1}} - b_{ss} \right),$$

(8)

where $\tau_{ss}$ and $b_{ss}$ are the steady state values of $\tau_t$ and $\frac{B_t}{Y_t}$, respectively. Importantly, $\gamma$ controls the speed of adjustment of the debt to output ratio towards its steady-state. Whenever $\gamma \neq 0$, we assume that lump-sum taxes, $T_t$, remain fixed at their steady state value, $T_{ss}$, compatible with $G_{ss}$, $\tau^c$, $\tau_{ss}$ and $b_{ss}$ (i.e., only the income tax is used to stabilize the debt-ratio). We also analyze the Ricardian version of the model, i.e., we set $\gamma = \rho = 0$ and assume the government balances the budget. Finally, the monetary authority sets the nominal interest rate according to a Taylor rule:

$$R_t = \max(Z_t, 1), \text{ where } Z_t = (\bar{\pi}_t - 1)\phi_n \ast \left( \frac{Y_t}{Y_{t-1}} - 1 \right) \phi_u.$$  

(9)

### 2.4 Market clearing

In equilibrium, all markets clear and the resource constraint, $Y_t = C_t + I_t + G_t + a (u_t) K_t$, completes the model.
3 Calibration, Simulations and Results

3.1 Parameters Choice

We calibrate the model for the U.S. economy, at quarterly frequency, by borrowing several values from existing literature. Following CER (2011), we set $\sigma_c = 2$, $\theta = 0.7$, $\alpha = 0.3$, $\delta_k = 0.02$, $\kappa = 17$, and $\phi_y = 0.25$. According to Uhlig (2010), we set $\sigma_L = 1$. $\chi$ is set such that, in steady state, $L_t$ is 0.31. Following Erceg et al. (2000), we set $\varepsilon_w = \varepsilon = 6$. $\Phi$ is set such that the profits-to-output ratio is 10% in the steady state, as in SGU (2006). Following SGU (2006), we set $\gamma_2 = 0.0685$ and $\iota^w = 1$. $\tau_{ss}$ is set to 0.2 which is roughly the mean of the tax rates on wages and capital as calibrated by Leeper et al. (2010), while $\tau_c$ is set to 0.028 following the same source. Following Traum and Yang (2011), we set $\rho = 0.92$. $G_{ss}$ is set such that the government consumption-to-output ratio in steady state is the average of the ratio in the post 1984 period, i.e., roughly 0.16. $b_{ss}$ is set such that the annualized government debt-to-output ratio is roughly that of the end of 2008, 0.65, when the nominal interest rate reached the ZLB. We set $\rho_{\lambda} = 0.5$. Finally, we set $\beta = 0.999$, $\xi_w = 0.75$, $\xi_p = 0.77$, $\iota^p = 0.66$, $\rho_G = 0.85$ and $\phi_\pi = 1.7$, which are values close to ones used in CER (2011) and SGU (2006).\footnote{The specific values used for this last set of parameters allow us to closely replicate the size of the multipliers obtained by CER (2011) in the specification with capital accumulation.}

3.2 The Experiment

In order to make the nominal interest attain the ZLB, we follow a strategy similar to CER (2011) and assume that the economy is in its steady state level in quarter 0. Then, we shock $\lambda_t$ at quarter 1, such that agents’ desire to save increases. We tune the shock such that, across all our simulations, the nominal interest rate hits the ZLB on impact and remains there for roughly 12 quarters. This generates a fall in aggregate demand, output and partly on prices. At quarter 1, we also generate an increase in government consumption of 1% of steady
state output.\textsuperscript{2} Then, we assume that $G$ follows a deterministic path, i.e., the autoregressive process described above without any uncertainty. We then calculate the (counterfactual) dynamic government-spending multipliers $t$ quarters after the increase in $G$ following:

$$M^ZLB_t = \frac{\sum_{k=0}^{t} (1 + r_{ss})^{-k} \left[ Y^{G,\lambda}_{k} - Y^{\lambda}_{k} \right]}{\sum_{k=0}^{t} (1 + r_{ss})^{-k} [G_{k} - G_{ss}]}$$

(10)

where $r_{ss}$ is the steady state real interest rate, $Y^{G,\lambda}$ is the output reaction to both the government and discount factor shocks whilst $Y^{\lambda}$ is the output reaction to the discount factor shock alone. We compute the perfect foresight solution of the model using the algorithm in Juillard (1996).

3.3 Results

We first analyze government-spending multipliers in the version of the model with fixed distortionary taxation, i.e., where only lump-sum taxes respond to the increase in $G$ while the income tax rate is fixed at its steady-state level. Under this Ricardian framework, we abstract from the dynamics of government debt. Panel 1 of Figure 1 shows dynamic output multipliers for some variations of this specification. The solid lines represent the output multipliers conditional on imposing substitutability between $C$ and $G$. These multipliers are obtained by setting $\phi$ and $\log(v)$ equal to 0.66 and 14.3, respectively, which are the values estimated in Ercolani and Valle e Azevedo (2012). The high value of $v$ implies that the aggregator in (1) becomes almost linear ($\tilde{C}_t \approx \phi C_t + (1 - \phi) G_t$), as in the specification estimated by Aschauer (1985) or Ahmed (1986). The dashed lines represent the output multipliers conditional on imposing separability between $C$ and $G$, i.e., setting $\phi = 1$. Focusing on the output multipliers at the ZLB, we note that the size of the multipliers generated by the

\textsuperscript{2}This simulated increase in $G$ is close to the maximum increase actually reached by government purchases as a result of the implementation of the American Recovery and Reinvestment Act. We are certainly aware that, as pointed out by CER (2011), the size of the multipliers depends on to the magnitude of the $G$ shock.
model with substitutability (i.e., the Subst NK ZLB line) is around two thirds of the one associated with the model with separabilities (i.e., the NK ZLB line), at any horizon.\(^3\) This reduction in the size of multipliers occurs even when the nominal interest is far above the ZLB (see the comparison between the NK and the Subst NK lines, and between the RBC and the Subst RBC lines).\(^4\)

![Figure 1](image)

**Figure 1**

**Multipliers/Responses in the Ricardian Model.** The lines are computed in the version of the model where only lump-sum taxes adjust to balance the budget, i.e., the income tax rate is fixed at its steady state level. Solid lines are obtained by imposing substitutability between \(C\) and \(G\). Dashed lines are obtained by imposing separability between \(C\) and \(G\). The lines labeled with ‘ZLB’ are counterfactual multipliers or responses, i.e. the difference between the multipliers/responses generated by the government consumption and the discount factor shocks and those generated by the discount factor shock alone (refer to equation (10)). The other lines are standard multipliers, calculated as explained in footnote 4. The x-axis is in quarters. The y-axis of Panel 3 measures the percentage deviation from the steady state.

Substitutability lowers output multipliers for two reasons. First, agents partially substitute private consumption with newly available government consumption. This depresses aggregate demand which, in the presence of nominal stickiness, translates into lower output, ceteris paribus. Notice that this effect tames the typically large increase in private aggregate demand generated by a government spending shock when the nominal interest rate is

\(^3\)Notice that the NK ZLB line closely reproduces the values of the multipliers found by CER (2011) in their specification with capital. Alike them, our impact multiplier is around 1.6 whilst the peak value is around 2.3.

\(^4\)Here, we must refer two things. First, the multipliers calculated when the economy is far above the ZLB are

\[
M_t = \sum_{k=0}^{1} (1+r_{ss})^{-k}[Y_k - Y_{ss}] / \sum_{k=0}^{1} (1+r_{ss})^{-k}[G_k - G_{ss}]
\]

where \(Y_{ss}\) and \(G_{ss}\) are output and government consumption at their respective steady state levels. Second, the RBC and the Subst RBC lines are generated by shutting down nominal rigidities in the model, i.e., by setting \(\xi_w, \xi_p, \iota^w\) and \(\iota^p\) to zero.
constrained at zero. Second, agents supply less labor in order to finance a lower level of consumption. These facts can be observed in the second and third panels of Figure 1, which shows the consumption multipliers and the reactions of labor in the separable government consumption world vis-à-vis the substitutability case, both at the ZLB.

Figure 2

**Multipliers in the Non-Ricardian Model.** The lines are computed in the version of the model where the fiscal rule is at work. Solid lines are obtained by imposing substitutability between $C$ and $G$. Dashed lines are obtained by imposing separability between $C$ and $G$. The lines labeled with 'ZLB' are counterfactual multipliers, i.e. the difference between the multipliers generated by the government consumption and the discount factor shocks and those generated by the discount factor shock alone (refer to equation (10)). The other lines are standard multipliers, calculated as explained in footnote 4. The x-axis is in quarters.

Figure 2 shows the output multipliers generated by the model with the fiscal rule at work, for three different speeds of debt adjustment. Panel 1 shows the multipliers generated by using $\gamma = 0.094$, estimated by Traum and Yang (2011). In this case, the debt-to-output ratio takes roughly 4 years to adjust (after the $G$ shock) prior to returning to its steady state. Focusing on the multipliers at the ZLB, we see that those produced within the separable government consumption world (i.e., the $NK \text{ ZLB}$ line) become negative after roughly 6 years, whilst the ones generated under substitutability (i.e., the $Subst NK \text{ ZLB}$ line) become negative on impact. These multipliers are smaller vis-à-vis their counterparts in Panel 1 of Figure 1. This occurs because, with fiscal rules operative, the government uses income taxes to dampen the expansion in debt, thereby discouraging agents from supplying labor and capital. For the sake of completeness, and borrowing the notation from Figure 1, Panel 1 also reports the multipliers generated by the model when the interest rate is far above the
ZLB. Panel 2 shows the same lines as Panel 1, but reports the case of a slower speed of debt adjustment. In this case, $\gamma$ is set to 0.06 such that the debt-to-output ratio takes roughly 12 years to reach the steady state after the $G$ shock. The output multipliers show dynamics similar to those observed in Panel 1, but, as expected, the size of the multipliers is larger at any horizon. Finally, Panel 3 shows the multipliers obtained in the case of a weak reaction of taxes to debt, i.e., setting $\gamma$ to 0.024. This value is such that the debt-to-output ratio takes roughly 25 years to reach the steady-state after the $G$ shock. Not surprisingly, these multipliers are closer to the ones presented in Panel 1 of Figure 1 (where only lump-sum taxes adjust).

4 Conclusions

Fiscal policy can be very effective in stimulating output when the nominal interest rate is at the ZLB. However, as CER (2011) has already pointed out, this mechanism is sensitive to the parametrization of the model. In this paper, we focus on one realistic channel that further questions the robustness of that result: the substitutability between private and government consumption. An aggressive debt-stabilizing fiscal rule can reinforce the negative effect on the size of the multipliers.

References


