The Costs of Nominal Public Debt in Bad Times: 
wealth and distortionary effects*†

José-Miguel Cardoso-Costa†
Portuguese Treasury and Government Debt Agency
Universidade Nova de Lisboa

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Abstract

Public debt denominated in nominal terms is costly when recessions are coupled with unexpectedly low inflation. I estimate the welfare costs of two large depression-deflation shocks: the US great depression and Japan’s lost decade. For a closed economy with 60% of public debt, these costs would be equivalent to a permanent consumption reduction of 2.27% and 0.46%, respectively. If, instead, this debt level was indexed to inflation, the welfare costs would be reduced by 0.34% and 0.12%. These figures only include the distortionary costs of holding nominal debt. In an economy with external nominal debt (credit), there is a positive (negative) wealth benefit of indexed debt to be added. For empirically relevant levels of public/external debt, however, the distortionary effect dominates.

JEL classification: H63; H21; E62; E44.
Keywords: nominal debt; indexed debt; deflation; distortionary costs.

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†The views expressed herein are those of the author and not necessarily coincident with those of the Portuguese Treasury and Government Debt Agency.

‡Faculdade de Economia, Universidade Nova de Lisboa, Campus de Campolide, 1099-032 Lisboa, Portugal. Email: jmccosta@fe.unl.pt. Web: http://docentes.fe.unl.pt/~jmccosta.
1 INTRODUCTION

The implications of having nominal or inflation-indexed debt for the conduct of monetary and fiscal policy have been examined at large, but there is a general perception that this discussion has fairly small welfare implications. This paper highlights instances in which the welfare difference of these two alternative debt denominations is not negligible, and thus suggests that one ought to be careful when downplaying the relevance of this discussion.

I concentrate on a costly event to an economy where the government holds only nominal debt, namely the case of a recession coupled with unexpectedly low inflation. I use two episodes from the past century to illustrate this idea – the great depression in the US, and the decade of long stagnation in Japan in the 90s – and ask the following question: what would be the welfare gain of having public debt indexed to inflation, instead of denominated in nominal terms, if an episode of this magnitude would occur in an economy with the current public debt levels?

Naturally, the idea that deflation hurts an economy that holds nominal debt, especially in a period of recession, is something that has been highlighted long ago, at least since Fisher’s (1933) description of his debt-deflation theory of great depressions. Fisher presents a striking figure: wholesale prices dropped 75% between 1929 and 1933, which implies that while in nominal terms US internal debt decreased by 20% in this period, in real terms debt actually increased some 40%.

He then goes on to explain why this may amplify a recession. Here, I will not deal with the possible implications of this fact for the duration or magnitude of the downturn. However, these numbers call attention to the potentially large wealth effect of having an indebted economy in nominal terms, when there are large price fluctuations. I disentangle this wealth impact from the distortionary effect of having public debt denominated in nominal terms.

I compare cases in which nominal government bonds are held by non-residents – so that deflation imposes a negative wealth shock in the economy –, with cases in which nominal government bonds are held by residents – and hence deflation is like a lump sum transfer from the government to the private sector. In the latter scenario welfare is only affected by the higher distortions needed to finance a larger real debt service.

In general, the wealth effect of deflation may be positive or negative, depending on

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1 Please refer to Cole and Ohanian (1999) and Cole, Ohanian and Leung (2005) for a thorough and modern discussion on this subject.
whether the economy is a net creditor or debtor (in nominal terms). It is trivial to show that the wealth effect is independent of the public debt level and proportional to the nominal net foreign asset position. On the contrary, the distortionary effect of deflation is always negative (for a positive level of public debt) and increases more than proportionately with the level of public debt. If all nominal public debt is held by non-residents, the wealth effect dominates, but for empirically relevant levels of the nominal external position, the distortionary effect dominates. Hence, our results suggest that the total welfare cost of holding nominal public debt in a deflationary period would always be negative and potentially significant.

The crucial assumption is thus the presence of distortionary taxes. This has long been introduced by Barro (1979) to show the optimality of tax smoothing over time. Lucas and Stokey (1983) formalize the idea in a micro-founded model, to show that taxes should be smoothed across states of nature. They highlight the relevance of using state-contingent assets in order to achieve this goal. More recently, Aiyagari et al. (2002) introduce incomplete markets in the Lucas and Stokey setting, recovering, at least partly, Barro’s unit root result. I set up an incomplete markets model similar to Aiyagari et al. (2002), but allow the agents to trade more financial instruments (namely a nominal bond). I do this in a small open economy model, in the spirit of Bohn (1990), where the real return of alternative financial instruments is given exogenously.

The baseline numerical exercise conducted within this framework suggests that, if a shock of the magnitude and length of the US great depression hits an economy with a public debt level equivalent to 60% of GDP, the welfare gain (distortionary effect) of having this debt indexed to inflation, instead of denominated in nominal terms, would be equivalent to a 0.34% consumption increase in every period. This compares with a welfare gain of 2.27% of completely eliminating the great depression.

This quantitative assessment underlines the potential costs of holding nominal debt. I thus hope that it helps to bring the discussion between nominal and inflation-indexed debt to the forefront of the more recent optimal public debt management literature.

In section 2 I set up a small open economy model, and the respective optimal taxation (Ramsey) problem, in a simplified version where all the uncertainty is resolved in the first period. Two alternative financial instruments may be available: (i) inflation-indexed (risk-free) bonds, and (ii) nominal bonds. In section 3 I describe preferences and the welfare measures used in the quantitative exercise. I take particular care in explaining how wealth and distortionary effects may be disentangled. In section 4 I calibrate the
shock to match the magnitude of the two historic episodes already referred: the US great depression and Japan’s lost decade. I then conduct the relevant welfare analysis. Section 5 concludes and discusses the relevance of the results.

2 MODEL

I consider an infinite-horizon small open economy without monetary policy. In this context, interest rates and inflation are exogenously given.\(^2\)

To simplify things, I assume that the real return on inflation-indexed bonds \(R^I\) is constant throughout the analysis, while the real return on nominal bonds \(R^N_t\) may be higher or lower than \(R^I\), if inflation is unexpectedly lower or higher, respectively. So, the real return on nominal bonds is stochastic. The economy is also subject to a productivity shock \(z_t\).

The purpose of this note is to quantify the welfare implications of having public debt denominated in real or nominal terms, in the event of a large unexpected shock. Hence, it is not instructive to simulate our economy with shocks drawn from a distribution used to represent “normal times”. Instead, I set up an example with uncertainty only in one period. In particular, I solve the optimal fiscal policy problem of this economy at period 0, knowing that in period 1 the state of the economy will be either Good or Bad. In the Bad state productivity is low, and the unexpected real return of nominal bonds is high. Each state occurs with probability \(p^i\), with \(\sum_{i=G,B} p^i = 1\). As I am mainly interested in tracing the effects of a Bad (and large) shock, the value of this probability is immaterial.

2.1 Households

The economy is populated by an infinitely-lived representative household, who seeks to maximize her expected lifetime utility:

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) = u(c_0, l_0) + \sum_{i=G,B} p^i \sum_{t=1}^{\infty} \beta^t u(c^i_t, l^i_t),
\] (1)

\(^2\)This may seem stringent at first, since one could argue that monetary policy could/should be used to revert a deflationary path – following Chari, Christiano and Kehoe (1991) --, especially in the wake of a recession. While this is certainly a possibility, I am not so much interested in what monetary policy could have done to revert the situation. Instead, my purpose here is to quantify the welfare implications of holding nominal public debt if such an event occurs, and it is undisputable that, either because central banks were unable, or unwilling, to proceed such a policy, such deflationary paths did occur in the past.
where \( u(c, l) \) is the period utility function, which is increasing in consumption \( c \), decreasing in labour \( l \), continuous, strictly concave, and satisfies the Inada conditions.

At period 0 the representative household faces the following budget constraint:

\[
c_0 + a_0^\Omega = (1 - \tau_0) z_0 l_0 + a_{-1}, \quad \Omega = I, N,
\]

where \( a_0^\Omega \) is the amount of bonds traded by the household at the end of period 0, which cost one unit of consumption in period 0. If the bond is indexed to inflation (\( \Omega = I \)) it pays a constant gross real interest rate of \( R_I \) in period 1. If the bond is denominated in nominal terms (\( \Omega = N \)) it pays a stochastic gross real return of \( R_N^{t} \), which is known only after the goods market opens in period 1.

Once the goods market opens in period 1, all uncertainty is resolved, so we can easily condense all future budget constraints in the following present value formulation\(^3\)

\[
\sum_{t=1}^{\infty} \beta^{t-1} \left[ c_i^t - (1 - \tau_t) z_t^t l_t^t \right] = A_1^{\Omega,i}, \quad \Omega = I, N, \quad i = G, B
\]

where \( A_1^{I,i} = R_I a_I^t \) if the agent trades an inflation-indexed bond, and \( A_1^{N,i} = R_N^t a_N^t \) if instead the bond is denominated in nominal terms.

In these constraints it is implicit the assumption of a linear production function (i.e. \( y_t = z_t l_t \)), which justifies the fact that the real wage equals the productivity level \( z_t \). \( \tau_t \) is a state-contingent labour income tax rate chosen by the government.

The first order conditions of the household’s problem are summarized by an intratemporal and an intertemporal conditions. The first is common to both specifications of the model and equates the marginal rate of substitution between consumption and leisure to the after tax wage income:

\[
(1 - \tau_t) z_t = -\frac{u_{t,c}}{u_{t,t}},
\]

where as usual \( u_i \) denotes the first derivative of \( u \) with respect to variable \( i \).

The second condition is an Euler equation and its particular form depends on the type of bond traded by the household. With inflation-indexed bonds we have

\[
u_{c,0} = \beta R_I^t E_0 [u_{c,1}] = \beta R_I^t \sum_{i=G,B} p_i^t u_{c,i}^t, \tag{5}\]

\(^3\)I am assuming the usual No Ponzi games condition \( \lim_{t \to \infty} \beta^{t+1} a_{t+1} = 0 \) and that asset prices are determined by a risk neutral outside agent with the same discount factor \( \beta \), so that \( R_t = 1/\beta, \forall t \geq 1 \).
and with nominal bonds

\[ u_{c,0} = \beta E_0 \left[ u_{c,1} R^N_1 \right] = \beta \sum_{i=G,B} p^i u_{c,1}^i R^{N,i}_1. \]  

(6)

From period 1 onwards, the intertemporal Euler equation is also common to both specifications, and equates the marginal utility of consumption over time:

\[ u_{c,t}^i = u_{c,t+1}^i, \quad \forall t \geq 1. \]  

(7)

2.2 Government

The government collects labour income taxes (which distort the intratemporal consumption-leisure choice) and issues new debt to pay for a fixed level of government expenditures \( g \) and for the service of public debt. Analogously to the household’s case the period-0 government’s budget constraint is given by:

\[ g + d_{-1} = \tau_0 z_0 \ell_0 + d_{0}, \quad \Omega = I, N. \]  

(8)

Again, from period 1 onwards we can condense all future government budget constraints in the following present value formulation

\[ \sum_{t=1}^\infty \beta^{t-1} \left[ \tau^i_t z^i_t \ell^i_t - g \right] = D_0^{\Omega,i}, \quad \Omega = I, N, \quad i = G, B. \]  

(9)

The natural framework to consider is one in which both households and the government borrow and lend the same type of bond. However, in an open economy it is also reasonable to have both agents trading with an external agent and not necessarily with each other, so that asymmetric settings are also possible. More importantly, the asymmetric cases help in disentangling the wealth and the distortionary effect of the shock. This will be further discussed in section 3.

As it is usual in a small open economy, asset prices are determined by a risk-neutral external agent,\(^4\) who trades both types of financial instruments. Then the following no-

\(^4\)The risk neutrality assumption stems from the fact that external agents are arguably able to diversify away all idiosyncratic risk.
arbitrage conditions arise:

\[ E_0 \left[ R_1^N \right] = \sum_{i=G,B} p^i R_1^{N,i} = R^I = \beta^{-1}. \] (10)

2.3 Ramsey problem

The competitive equilibrium of the economy is defined by the first order conditions of the household’s problem and the government budget constraint. This will naturally depend on the type of financial instruments available. I describe here the Ramsey problem for the case where both the consumer and the government trade a nominal bond.\(^5\)

As usual in this literature, it is useful to recast this problem using the primal approach of Lucas and Stokey (1983). Since the interest rates are defined exogenously, in our case we just need to substitute the tax rate in both the household’s and government’s budget constraints, using (4). Then the Ramsey problem is characterized only on the allocations of consumption, labour, private assets and public debt:

\[
\begin{align*}
\max_{\{c_t, l_t, a_{N,t}, d_{N,t}\}} & \quad E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \\
\text{s. to} & \quad c_0 + a_{0,N} = a_{-1} - \frac{u_{t,0}}{u_{c,0}} l_0 \quad (12) \\
& \quad g + d_{-1} = \left( z_0 + \frac{u_{t,0}}{u_{c,0}} \right) l_0 + d_{0,N} \quad (13) \\
& \quad u_{c,0} = \beta \sum_{i=G,B} p^i u_{c,i}^{N,i} R_1^{N,i} \quad (14) \\
& \quad \sum_{t=1}^{\infty} \beta^{t-1} \left[ c_t^i + \frac{u_{i,t}^i}{u^c_{i,t}} l_t^i \right] = R_1^{N,i} a_{0,N}^i, \quad i = G, B \quad (15) \\
& \quad \sum_{t=1}^{\infty} \beta^{t-1} \left[ (z_t^i + \frac{u_{i,t}^i}{u^c_{i,t}}) l_t^i - g \right] = R_1^{N,i} d_{0,N}^i, \quad i = G, B \quad (16) \\
& \quad u_{c,t}^i = u_{c,t+1}^i = K^i, \quad \forall t \geq 1, i = G, B \quad (17) \\
& \quad a_{-1}, d_{-1} \text{ given.}
\end{align*}
\]

The corresponding first order conditions are relegated to Appendix A. The assumptions on the utility function guarantee that these conditions are necessary and sufficient for a maximum. This system of equations can then be solved to obtained the exact solution of

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\(^5\)The cases with indexed debt follow directly replacing \( R_1^N \) by \( R^I \) where appropriate.
this optimal taxation problem.

I will now describe preferences and discuss the welfare measures used, in particular the proposed method to disentangle between wealth and distortionary effects.

3 PREFERENCES AND WELFARE MEASURES

I use GHH preferences for the numerical exercise

\[ u(c, l) = \frac{(c - \eta l^{\xi})^{1-\sigma} - 1}{1 - \sigma}. \]  

(18)

It is now common to use this type of utility function in small open economy settings, as it seems necessary to obtain reasonable dynamics of consumption and output.\(^6\) Moreover, this choice also facilitates the interpretation of our welfare measures.

Throughout the analysis, I will mostly concentrate on the welfare obtained in the Bad state, as the purpose is precisely to understand how the distinction between nominal and indexed debt may be relevant under such an event. I will in any case assume a small probability for the occurrence of this event, and also provide a measure of expected welfare.

In all cases, I present consumption equivalent welfare differences, as proposed by Lucas (1987). These are computed against a status quo, which is taken to be an economy with nominal assets on both sides of the economy (private and public sectors). Our numbers, however, are not directly comparable with Lucas’s, mainly because the agents in our economy draw utility from leisure, so that recessions that reduce the number of hours worked are not as bad. Also, the simplified version of the model with uncertainty in only one period may lead to a different magnitude of the consumption equivalent welfare measure.

Hence, in order to get a proper benchmark, I will also compute the welfare gain of completely eliminating fluctuations (as Lucas). Interestingly, for the preferences specified above, the welfare under certainty is indistinguishable from that achieved when all agents are allowed to trade a complete set of state-contingent assets.

\(^6\)See in particular Correia, Neves and Rebelo (1995). The solution of the optimal policy problem presented here confirms the idea that, in a small open economy setting, GHH preferences perform much better in reconciling consumption volatility obtained in the model, with that observed in the data. In all the examples shown below, the relative variability of consumption, against that of output, is between 0.7 and 0.9. Using several parameterizations of the more usual iso-elastic preferences, I was never able to obtain a number above 0.3, which is clearly inconsistent with the data.
As lemma 1 in appendix B shows, households have access to such risk-sharing technology, welfare in each state is independent from the realization of the stochastic process. Welfare in this case is given by

\[ E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) = \frac{K^{\frac{\sigma-1}{\sigma}} - 1}{(1 - \beta)(1 - \sigma)}, \]  

(19)

where \( K = u_c(c_t, l_t), \forall t \geq 0, \forall i,j \). Depending on the financial structure faced by the government, however, \( K \equiv u_{c,t} \) may be different, so that welfare will in general differ for alternative government market structures.

Nevertheless, when the government has access to complete markets, I find that the difference between the case with and without fluctuations is numerically indistinguishable. Thus, I will refer to the welfare gain of moving to complete markets or eliminating fluctuations interchangeably.

3.1 Disentangling wealth and distortionary effects

As already discussed, it will also be crucial to disentangle between wealth and distortionary effects. Lemma 2 stated also in appendix B proves to be useful for this purpose. It shows that, in the special case of GHH preferences labour supply is not affected by wealth shocks.

In order to apply this result, note that in our case the wealth effect is only present when the net foreign asset position is exposed to inflation risk, e.g. because some nominal public bonds are being held by non-residents. To see this start from the optimal allocation obtained in the economy with only indexed assets (denote this by \( x_{1,i} \)), but assume now that the assets/debt positions were denominated in nominal terms. Now, assume also that the shock on the real return of nominal government bonds – and only that amount – can be paid with lump sum taxes. In this case the household’s and government’s budget constraints at \( t = 1 \) are

\[ \sum_{t=1}^{\infty} \beta^{t-1} \left[ c_t^{I,i} - (1 - \tau_t^{I,i}) z_t^{I,i} l_t^{I,i} \right] = R_{1,i}^{N,i} d_0^N - T_1, \]

\[ \sum_{t=1}^{\infty} \beta^{t-1} \left[ \tau_t^{I,i} z_t^{I,i} l_t^{I,i} - g \right] + T_1 = R_{1,i}^{N,i} d_0^N, \]
where \( T_i \equiv \left( R_{1,i}^N - R^t \right) d_0^N \).

Then, the government budget constraint is trivially satisfied, and households suffer a wealth shock of \( \left( R_{1,i}^N - R^t \right) a_0^N - T_i = \left( R_{1,i}^N - R^t \right) (a_0^N - d_0^N) \). It is thus trivial to understand that there is no wealth effect when \( a_0^N = d_0^N \) (i.e. when the net external position of the country is not exposed to inflation risk). An unexpectedly low inflation raises the real cost of servicing public debt, but has a compensating positive effect on the real return of these bonds. If the households in the economy are holding these bonds we implicitly have a lump-sum transfer from the government to the private sector that shuts down an overall wealth effect on the economy. In that scenario, any welfare costs of holding nominal public debt only capture the distortionary effects of having to change the tax rate to pay for higher costs of servicing public debt.

If \( a_0^N \neq d_0^N \), however, households will change their consumption and labour decisions, which would in general affect the whole optimal allocation. As Lemma 2 shows, however, under GHH preferences labour supply is not affected by wealth shocks, so that all the burden of a wealth shock lies on consumption. Then, under the wealth shock considered, consumption changes by the constant amount \( \frac{1-\beta}{\beta} \left( R_{1,i}^N - R^t \right) (a_0^N - d_0^N) \) in every period, for a given state \( i \), which clarifies our statement that the wealth effect of holding nominal public debt is proportional to the nominal net foreign asset position \( a_0^N - d_0^N \). I take the consumption equivalent gain of the allocation obtained after this wealth shock, relative to the optimal allocation under indexed debt, as a measure of the wealth cost of holding nominal public debt. The distortionary effect will then be computed as the difference between the total welfare gain and the welfare change that can be imputed to the wealth shock.

4 RESULTS

The parameterization of the utility function follows directly from the literature. As in Correia, Neves and Rebelo (1995), I assume \( \sigma = 2 \), and \( \xi = 1.7 \). Then, \( \eta \) is chosen to match a labour supply of \( \frac{1}{3} \) of the time, in an economy with no initial public debt.

In all cases the time period of the model is 5 years. This choice is not so much driven by the length of the shocks considered, but more by the maturity of government debt. According to OECD data, in the past couple of decades the average term to maturity of public debt has been around 5 years, both in US and Japan, and also in an average OECD country. Given the nature of the unexpected shock to the real return of nominal bonds,
this should affect the value of bonds issued before uncertainty is resolved. Once the shock is known, new bond issues will incorporate this information, so that the nominal interest rate would adjust to guarantee a real return of nominal bonds in line with that of indexed bonds.

I assume a constant annual real return of inflation-indexed bonds equal to 2.5%, so that \( R^I = 1.125 \). The baseline public debt level is fixed at 60% of total income. This is roughly the average level observed in OECD countries today, and it is also very close to the level observed in Japan in the end of 1990. I start by assuming the natural case of a balanced foreign asset position, again in order to shut down overall wealth effect, so the amount of private assets is also 60% of GDP in the baseline case. I then solve the model for a wide range of public debt levels. In all cases, government expenditures are fixed at 20% of GDP.

I will now calibrate the values of \( z \) and \( R^N \) in period 1 to match two historic episodes of a recession coupled with deflation: the US in the 30s and Japan in the 90s. It is perhaps important to stress at this point that this exercise is not intended to exactly replicate the dynamics of consumption or labour supply (or even of productivity or unexpected inflation) in either of these periods. Remember that the objective is to obtain, in the simplest possible setting, an estimate of the welfare differences of having nominal or inflation-indexed debt. Thus, in the calibration, I am merely interested in getting a sense of the order of magnitude of these shocks.

4.1 US great depression

The US great depression started in the Autumn of 1929, so I place ourselves in the end of that year. For the productivity shock I use Cole and Ohanian’s (1999) detrended TFP measure. In the 5 years between 1929 and 1934 this has been on average 9% below trend. I thus take \( z^B_1 = 0.91 \). In the same period, prices have dropped around 25% (on average 5% per year). Thus, I take \( R^{N,B}_1 = 1.125 + 0.25 = 1.375 \).

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7 The average real long-term interest rate (deflated with CPI inflation) was 2.7% in the US between 1960 and 2006. In Japan it was 3% between 1980 and 2006.

8 In US or the euro area the net foreign asset position is about −15% of GDP, while in Japan it is around 40%. I will then use these two alternative values to obtain reasonable quantitative measures of the wealth effect.

9 I am using CPI data from the Bureau of Labor Statistics that starts in 1913.

10 I have also estimated a VAR using both inflation and Maddison’s measure of GDP per capita (available from http://www.ggdc.net/maddison/) for the sample 1913-1929, which then enables me to compute the unexpected component of inflation for the 1930-1934 period. In that period, inflation was on average 10
I assume that the probability of a Bad state is only 1%. The values of $z$ and $R^N$ in the Good state are computed so that $E[z_1] = \bar{z} = 1$ and $E[R^N_1] = R^I$.

**Effects of a bad shock.**

Figure 1 in Appendix C shows the effects of the Bad shock on consumption, labour, the tax rate, and the market value of assets and public debt under the two symmetric market arrangements (indexed and nominal). These are like the impulse response functions to an i.i.d. shock, with the magnitude of that occurred in 1929-34.

The figure also includes the impulse responses of an economy where both agents have access to complete markets. As one can see, under such framework, the tax rate is perfectly smoothed out. Moreover, both consumption and labour decrease on impact, but only temporarily. Hence, the optimal allocation inherits the statistical properties of the underlying shock, a feature that would be expected in a complete markets setting, as in Lucas and Stokey (1983). The market value of the assets/debt is unchanged.

Under incomplete markets, however, even this temporary shock has permanent effects. Consumption decreases permanently, while the tax rate must be raised. Private and public debt increase, both contributing to a permanent reduction in national savings.\(^{11}\)

Crucially, the difference is also significant when compared the nominal and the inflation-indexed debt cases, the responses being much larger when the economy has all its bonds denominated in nominal terms. In any case, the evolution of the net foreign asset position is similar in both cases.

**Welfare analysis.**

I start by presenting the welfare gain of completely eliminating this fluctuation, which as described above is equal to allowing both agents to trade under complete markets. This welfare gain is equivalent to a permanent increase in consumption of 2.27%, relative to the status quo case of an economy with both nominal assets and public debt.

Table 1 shows the analogous consumption equivalent welfare gains of each alternative setting. For the sake of completeness, I present results for all possible 4 combinations of percentage points below what would be expected using data up until the end of 1929. Using this instead would further raise the cost of nominal debt.\(^{11}\)

\(^{11}\)These responses are in line with those described by Marcet and Scott (2009) for a closed economy. The main difference in this small open economy framework is the response of the tax rate, which is perfectly smoothed out in the complete markets case and is a perfect unit root in the incomplete markets case. This is a particular feature of GHH preferences. Under additively separable utility functions, this kind of response would be reflected on consumption, instead.

\[12\]
Table 1. Welfare gains: US great depression shock

<table>
<thead>
<tr>
<th>Households’ asset denomination</th>
<th>Public debt denomination</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nominal</td>
</tr>
<tr>
<td>Nominal</td>
<td>0.00%</td>
</tr>
<tr>
<td>Indexed</td>
<td>-0.42%</td>
</tr>
</tbody>
</table>

Legend: Consumption equivalent gain relative to the Nom/Nom case, when a bad state similar to the US great depression between 1929 and 1934 occurs. Public debt to GDP ratio is 60%, and the net foreign asset position is balanced.

nominal and indexed assets.

The welfare gain of moving towards the symmetric indexed assets/debt case is equivalent to a consumption increase of 0.34% in every period. Notice that, as explained above, this includes only the distortionary costs of nominal public debt, since all nominal debt is held by domestic households. This means that 15% of the distortionary welfare cost of holding 60% of nominal public debt in the Bad state would be absorbed if this debt was indexed to inflation.

Rectangles and triangles.—

Now, if all nominal government debt was held by non-residents (a scenario captured by the indexed assets / nominal debt case in Table 1), the welfare cost of nominal public debt would be further raised by 0.42%.

This is a measure of the wealth cost, which is unsurprisingly dominant when nominal public debt is equal to nominal external debt (“the rectangle is larger than the triangle”). However, for the net foreign asset positions observed in most developed countries it is possible that the distortionary effect dominates (in this case the wealth effect is only "part of the rectangle").

In order to address this issue, I present the welfare analysis for alternative levels of public debt and the net foreign asset position. This gives a sense of how the welfare costs depend on these variables, and crucially on how the distortionary component depends on the level of public debt.

I focus here only on the gain between the two symmetric cases (all nominal to all...
Table 2. Welfare gains with net foreign asset position of -0.15

<table>
<thead>
<tr>
<th>Public debt to GDP ratio</th>
<th>30%</th>
<th>60%</th>
<th>90%</th>
<th>120%</th>
<th>150%</th>
<th>180%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>wealth effect</strong></td>
<td>0.114%</td>
<td>0.117%</td>
<td>0.122%</td>
<td>0.127%</td>
<td>0.134%</td>
<td>0.143%</td>
</tr>
<tr>
<td><strong>total welfare gain</strong></td>
<td>0.241%</td>
<td>0.445%</td>
<td>0.749%</td>
<td>1.232%</td>
<td>2.110%</td>
<td>4.340%</td>
</tr>
<tr>
<td><strong>distortionary effect</strong></td>
<td>0.127%</td>
<td>0.328%</td>
<td>0.627%</td>
<td>1.105%</td>
<td>1.976%</td>
<td>4.197%</td>
</tr>
</tbody>
</table>

For comparison...

| welfare of eliminating shock | 2.062% | 2.398% | 2.874% | 3.596% | 4.838% | 7.743% |
| relative welfare gain        | 12%    | 19%    | 26%    | 34%    | 44%    | 56%    |

Legend: Consumption equivalent gains of Ind/Ind case, relative to the Nom/Nom case, when a bad state similar to the US great depression between 1929 and 1934 occurs.

Indexed gain), and also compare this with the gain of eliminating the fluctuation. Wealth effects are also present because the net foreign asset position is unbalanced. Tables 2 and 3 present the results for a net foreign asset position of −15% and 40%,\(^\text{13}\) respectively, and a wide range of public debt levels.\(^\text{14}\)

For a given nominal net foreign asset position, I find that the wealth effect, in consumption equivalent terms, is relatively small and broadly independent on the level of public debt.\(^\text{15}\) On the contrary, the distortionary effect increases exponentially with public debt, and overcomes the wealth effect even for moderate levels of public debt.

These figures can again be compared with the welfare gain of eliminating the fluctuations. As one can observe, the welfare gain of indexing public debt is relatively more important as the public debt level increases. For an economy with 180% of public debt, having indexed debt in this scenario could absorb 50% of the welfare cost of a large fluctuation. Notice that this is true even if the economy holds a positive nominal external position (as again is the case of Japan), where deflation has a positive wealth effect.\(^\text{16}\)

\(^\text{13}\)These numbers were chosen to match the current external position of US and Japan, respectively, which correspond to the more extreme positions observed among developed countries.

\(^\text{14}\)The range of public debt levels considered is intended to capture the wide dispersion observed in reality, where again Japan stands out with a public debt level of around 180% of GDP.

\(^\text{15}\)As one may observe, the wealth effect in consumption equivalent terms is not exactly independent on the level of public debt. The small differences observed stem from different steady state levels of consumption, which are smaller when the public debt to GDP ratio is larger (higher debt service costs impose larger distortions).

\(^\text{16}\)In table 3 the wealth benefit of holding a positive nominal external position, is captured by the
Table 3. Welfare gains with net foreign asset position of 0.4

<table>
<thead>
<tr>
<th>Public debt to GDP ratio</th>
<th>30%</th>
<th>60%</th>
<th>90%</th>
<th>120%</th>
<th>150%</th>
<th>180%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>wealth effect</strong></td>
<td>-0.296%</td>
<td>-0.305%</td>
<td>-0.317%</td>
<td>-0.330%</td>
<td>-0.347%</td>
<td>-0.369%</td>
</tr>
<tr>
<td><strong>total welfare gain</strong></td>
<td>-0.129%</td>
<td>0.055%</td>
<td>0.332%</td>
<td>0.778%</td>
<td>1.595%</td>
<td>3.687%</td>
</tr>
<tr>
<td><strong>distortionary effect</strong></td>
<td>0.167%</td>
<td>0.361%</td>
<td>0.649%</td>
<td>1.108%</td>
<td>1.942%</td>
<td>4.056%</td>
</tr>
</tbody>
</table>

For comparison...

| welfare of eliminating shock | 1.652% | 1.964% | 2.406% | 3.081% | 4.247% | 6.985% |
| relative welfare gain        | -8%    | 3%     | 14%    | 25%    | 38%    | 53%    |

Legend: Consumption equivalent gains of Ind/Ind case, relative to the Nom/Nom case, when a bad state similar to the US great depression between 1929 and 1934 occurs.

**Expected welfare gains.**

I finish the analysis by contrasting these numbers with the expected welfare gains at $t = 0$. These numbers are shown in tables 5 and 6 in Appendix C. Given the perfect negative correlation between $z$ and $R^N$, we would expect indexed public debt to be preferable, which is indeed the case. However, the welfare differences are naturally much smaller, because in the Good state nominal debt would have a small welfare gain, and the probability of the Bad state occurring is very small. These numbers are in line with previous estimates, e.g. Aiyagari et al. (2002) estimate a welfare gain of moving from incomplete to complete markets of 0.0092%.

More importantly, the relative welfare gain of having indexed debt, when compared to the welfare of completely eliminating the fluctuations, is still significant, and actually larger than what we found in the Bad state.

4.2 Japan’s lost decade

For the technology shock I use the TFP series from the AMECO database. I detrend this using the average growth rate between 1970 and 1990. I estimate that TFP was on average 1.6% below this trend in the period 1991-1995, so I take $z_1^H = 0.984$. The shock to unexpected inflation is the difference between observed inflation and the forecast from a VAR estimated using TFP and CPI from 1960 to 1990. The unexpected component negative wealth effect of having assets/debt indexed to inflation.
Table 4. Welfare gains: Japan’s lost decade shock

<table>
<thead>
<tr>
<th>Households’ asset denomination</th>
<th>Public debt denomination</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>Nominal Indexed</td>
</tr>
<tr>
<td>0.00%</td>
<td>0.28%</td>
</tr>
<tr>
<td>Indexed</td>
<td>-0.16%</td>
</tr>
<tr>
<td></td>
<td>0.12%</td>
</tr>
</tbody>
</table>

Legend: Consumption equivalent gain relative to the Nom/Nom case, when a bad state similar to the Japan’s stagnation between 1991 and 1995 occurs. Public debt to GDP ratio is 60%, and the net foreign asset position is balanced.

of inflation for the subsequent 5 years was estimated at 2% per year. Thus, \( R_{1}^{N,B} = 1.125 + 0.02 \times 5 = 1.225. \)

Though in a smaller scale, the impulse responses of the optimal allocation to this shock are naturally very similar to the ones observed in the US, so I omit them from here. The welfare gains for an economy with a public debt to GDP ratio of 60% and a balanced external position is presented in table 4.

The magnitude of the welfare gains is naturally smaller, but when contrasted with the welfare gain of eliminating this shock (0.46%), we draw a similar conclusion from that with the US great depression: in an economy with a public debt to GDP ratio of 60%, having public debt indexed to inflation would absorb some 25% of the welfare cost of a shock analogous to that affecting Japan in the 90s.

5 CONCLUSIONS

This paper calls attention to the discussion between nominal and inflation-indexed public debt, by providing a quantitative assessment of the welfare costs that arise in a deflationary episode.

This distinction is particularly relevant today as we face a serious recession coupled with, at least, a non-negligible risk of a deflationary period. Further fiscal stimulus that raise government expenditures would add even more pressure to an economy with nominal debt (unless such a policy also translates in higher inflation), but the numbers shown here abstract from such policy measures.

The main purpose of this note is to measure the welfare difference between nominal and indexed debt, when monetary policy is unable, or unwilling, to avoid such a deflationary
path. In that regard, analyzing a deflationary period – where nominal debt is a burden –, or an inflationary one – where nominal debt is a blessing –, would be indifferent.\textsuperscript{17} Nevertheless, given the difficulty in anticipating the correct correlation between inflation and other sources of uncertainty in reality, the magnitude of the welfare differences shown here is itself an argument in favour of at least more indexation than the level observed today in most countries.

The episodes analyzed here are analogous to the situation of emerging economies that enter a financial crisis with most of its liabilities denominated in a foreign currency. Once the domestic currency starts to depreciate, the value of its liabilities may increase dramatically, putting under even more pressure the solvency of the country.

In our case, it is deflation (or unexpectedly low inflation) that puts the fiscal position under pressure, in an economy already hit by a recession. However, while for emerging economies it may be difficult to find investors willing to lend in domestic currency, in the case of developed countries there is today not a clear reason for avoiding the issuance of instruments indexed to inflation. In fact, while a couple of decades ago it was argued that markets for these instruments were not sufficiently established, so that a liquidity premium had to be paid when issuing inflation-indexed bonds, the success of the development of these markets in the past 20 years, notably in the UK, has somewhat contradicted this view.

In recent years a number of countries, namely in the euro area, have exactly moved in this direction and started issuing more inflation-indexed bonds. This should be welcomed in light of the results presented here.

\textsuperscript{17}The stagflation period of the 70s would be a good example to argue in favour of nominal debt. However, this period also highlights the risks faced by nominal bond holders. As Campbell, Sundaram and Viceira (2009) show, when the correlation between the economic cycle and inflation turns negative (as in the 70s) the risk premium of nominal bonds increases. Here I assume there is no risk premium on nominal bonds, otherwise the cost of nominal debt would be even larger.
Appendix A – Lagrangean and first order conditions of the Ramsey problem

Take the Ramsey problem in (11). The last constraint of this problem implies that the marginal utility of consumption is constant from period 1 onwards. I use this equation \( u_c(c_i^t, l_i^t) = K^i \) to define the labour supply as a function of consumption and marginal utility \( K^i \). Hence, when writing the Lagrangean function of this problem I omit the last constraint, and use the substitution \( l_i^t = l(c_i^t, K^i) \), \( \forall t \geq 1 \). Then \( K^i \) must be included in the set of control variables, instead of \( l_i^t \), \( \forall t \geq 1 \).

I also define \( i_t = u_l(l_t; K_t) \) to simplify a bit the expressions.

\[
L = u(c_0, l_0) + \sum_{t=1}^{\infty} \beta^t \sum_{i=G,B} \beta^i u(c_i^t, l(c_i^t, K^i)) + \\
+ \psi_0 \left[ c_0 + \chi_0 l_0 + a_0^N - a_{-1} \right] + \lambda_0 \left[ (z_0 + \chi_0) l_0 - g + d_0^N - d_{-1} \right] + \\
+ \beta \sum_{i=G,B} \rho^i \left[ \sum_{t=1}^{\infty} \beta^{t-1} \left( (c_i^t + \chi_i l(c_i^t, K^i)) - R_t^{N,i} a_0^N \right) \right] + \\
+ \beta \sum_{i=G,B} \rho^i \chi^i \left[ \sum_{t=1}^{\infty} \beta^{t-1} \left( (z_i^t + \chi_i) l(c_i^t, K^i) - R_t^{N,i} a_0^N \right) \right] + \\
+ \gamma \left[ u_{c,0} - \beta \sum_{i=G,B} \rho^i K^i R_t^{N,i} \right].
\]

The first order conditions are then the following

\[
u_{c,0} + \psi_0 \left( 1 - \chi_{c,0} l_0 \right) - \lambda_0 \chi_{c,0} l_0 + \gamma u_{c,c,0} = 0,\\u_{t,0} + \psi_0 \left( \chi_{t,0} l_0 + \chi_0 \right) + \lambda_0 \left( z_0 + \chi_{t,0} l_0 + \chi_0 \right) = 0,\\u_{c,t} + u_{t,0} l_{c,t} + \psi_i + \left( \psi^i + \lambda^i \right) \left( \chi^i_{c,t} l_{t} + \chi^i_{c,t} \right) + \lambda^i z_{t}^{i,i} = 0, \forall i = G,B, t \geq 1,\\\sum_{t=1}^{\infty} \beta^{t-1} \left[ u^i_{t,0} l^i_{K,t} + \left( \psi^i + \lambda^i \right) \left( \chi^i_{t} l^i_{t} + \chi^i_{t} \right) l^i_{K,t} + \lambda^i z_{t}^{i,i} \right] - \gamma R_t^{N,i} = 0, \forall i = G,B,\\\psi_0 - \beta \sum_{i=G,B} \rho^i \psi_i R_t^{N,i} = 0,\\\lambda_0 - \beta \sum_{i=G,B} \rho^i \lambda^i R_t^{N,i} = 0.
\]

\(^{18}\)If the utility function is additively separable in consumption and labour, the last constraint of the Ramsey problem implies that consumption is constant from period 1 onwards. Hence, when writing the Lagrangean function of this problem I omit this last constraint, and simply use the substitution \( c_i^t = c_i^t, \forall t \geq 1 \).
Appendix B – Some results with GHH preferences

This Appendix states and proves two properties of the optimal allocation under GHH preferences.

The first property uses the last constraint in the Ramsey problem (17) to show that in this case welfare will be constant over time and across states.

Lemma 1 If preferences are of the GHH form given by (18) and if households have access to a complete set of state contingent assets with prices given by the risk-neutral probability measure, welfare is given by

\[ u(c_t, l_t) = \frac{K^{\frac{\sigma-1}{\sigma}} - 1}{1 - \sigma}, \]  

where \( K = u_c(c_t, l_t) \), \( \forall_{t \geq 0, s} \). Moreover, \( \frac{\partial V}{\partial K} < 0 \).

Proof. The proof is straightforward. If households have access to a complete set of state contingent assets, with prices given by the risk-neutral probability measure, the intertemporal first order conditions of the household’s problem (17) can be used to define consumption as a function of labour supply and the (constant) marginal utility of consumption \( K \). With GHH preferences:

\[ u_c(c_t, l_t) = (c_t - \eta_l^t)^{-\sigma} = K \]
\[ c_t = K^{-\frac{1}{\sigma}} + \eta_l^t. \]  

Substituting this in the utility function we obtain (20). □

Note that this property depends only on allowing households to have access to complete markets, i.e. it would still hold even when the government is constrained to an incomplete markets setting. However, depending on the financial structure faced by the government, \( K \equiv u_c(t, l) \) may be different. In particular, if the government does not have access to complete markets, it may have to change the tax rate more than under complete markets, which may lead to a different relation between consumption and labour supply, and hence different marginal utility of consumption (i.e. different \( K \)). While it would still be true that, for that specific economy, agents would be indifferent to fluctuations, the (constant) welfare will in general differ for alternative government market structures.

The second property is an important result on the effects of a wealth shock under GHH preferences. This result allows us to disentangle between wealth and distortionary implications of having different public debt instruments, as is described in section 3.

Lemma 2 Under GHH preferences, any wealth shock \( \Delta W_t \) faced by the households in period \( t \) (i) does not affect labour supply. Furthermore, if households have access to a
complete set of state contingent assets with prices given by the risk-neutral probability measure, then any wealth shock \( \Delta W_t \) (ii) affects \( c_{t'} \) in every period \( t' \geq t \) by the same amount, given by \( (c_{t'}^W - c_{t'}) = \frac{1 - \beta}{\beta} \Delta W_t \).

**Proof.** Part (i) follows directly from the household’s intratemporal first order condition (4) under GHH preferences:

\[
(1 - \tau_t)z_t = -\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = \eta \xi_l^{t-1}.
\] (22)

As we can observe, labour supply is completely determined by the tax rate and the productivity level, and not by the level of wealth. Then, the whole burden of any wealth shock falls on consumption.

If households have access to complete markets, the difference between the budget constraints, before and after the wealth shock in period \( t \), is given by

\[
E_t \sum_{t'=t}^{\infty} \beta^{t'-t} [c_{t'}^W - c_{t'}] = \Delta W_t,
\]

where \( c_{t'}^W \) and \( c_{t'} \) are, respectively, consumption allocations with and without the shock. Since from the intertemporal first order conditions we have \( u_c(c_t, l_t) = K, \forall t, s \), the effect of the wealth shock on consumption \( (c_{t'}^W - c_{t'}) \) must be evenly spread through all periods. Thus, the budget constraint further simplifies to

\[
\frac{\beta (c_{t'}^W - c_{t'})}{1 - \beta} = \Delta W_t.
\]
Legend: Responses to a negative productivity shock, coupled with unexpectedly low inflation, similar to the US great depression between 1929 and 1934. Three alternative symmetric asset market structures are represented: nominal bonds (Nom/Nom), inflation-indexed bonds (ILB/ILB), and complete markets (CM/CM). Public debt / GDP = 60%. Net foreign asset position = 0.
Table 5. Expected welfare gains with net foreign asset position of -0.15

<table>
<thead>
<tr>
<th>Public debt to GDP ratio</th>
<th>30%</th>
<th>60%</th>
<th>90%</th>
<th>120%</th>
<th>150%</th>
<th>180%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>wealth effect</strong></td>
<td>0.0001%</td>
<td>0.0001%</td>
<td>0.0001%</td>
<td>0.0002%</td>
<td>0.0002%</td>
<td>0.0002%</td>
</tr>
<tr>
<td><strong>total welfare gain</strong></td>
<td>0.0004%</td>
<td>0.0009%</td>
<td>0.0018%</td>
<td>0.0037%</td>
<td>0.0082%</td>
<td>0.0239%</td>
</tr>
<tr>
<td><strong>distortionary effect</strong></td>
<td>0.0003%</td>
<td>0.0007%</td>
<td>0.0017%</td>
<td>0.0035%</td>
<td>0.0080%</td>
<td>0.0237%</td>
</tr>
</tbody>
</table>

For comparison...

| welfare of eliminating uncertainty | 0.0014% | 0.0020% | 0.0032% | 0.0054% | 0.0104% | 0.0273% |
| relative welfare gain              | 27%     | 43%     | 57%     | 69%     | 79%     | 88%     |

Legend: Consumption equivalent gain relative to the Nom/Nom case, when a bad state similar to the US great depression between 1929 and 1934 occurs with 1% probability.

Table 6. Expected welfare gains with net foreign asset position of 0.4

<table>
<thead>
<tr>
<th>Public debt to GDP ratio</th>
<th>30%</th>
<th>60%</th>
<th>90%</th>
<th>120%</th>
<th>150%</th>
<th>180%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>wealth effect</strong></td>
<td>-0.0003%</td>
<td>-0.0003%</td>
<td>-0.0003%</td>
<td>-0.0004%</td>
<td>-0.0004%</td>
<td>-0.0005%</td>
</tr>
<tr>
<td><strong>total welfare gain</strong></td>
<td>0.0000%</td>
<td>0.0004%</td>
<td>0.0012%</td>
<td>0.0029%</td>
<td>0.0070%</td>
<td>0.0213%</td>
</tr>
<tr>
<td><strong>distortionary effect</strong></td>
<td>0.0003%</td>
<td>0.0007%</td>
<td>0.0016%</td>
<td>0.0033%</td>
<td>0.0074%</td>
<td>0.0218%</td>
</tr>
</tbody>
</table>

For comparison...

| welfare of eliminating uncertainty | 0.0009% | 0.0015% | 0.0025% | 0.0045% | 0.0091% | 0.0245% |
| relative welfare gain              | 1%     | 28%     | 49%     | 65%     | 77%     | 87%     |

Legend: Consumption equivalent gain relative to the Nom/Nom case, when a bad state similar to the US great depression between 1929 and 1934 occurs with 1% probability.
REFERENCES


