Abstract

This paper uses a citizen-candidate model in a two-layer government setup to analyze how the assignment of tasks between the central and local governments endogenously determines the quality of politicians at both levels. We setup a model where a political job is composed of several tasks, and the outcome of each task is a random variable with a higher expected value, the more competent is the politician that performs it. Voters observe the outcome of the different tasks but not the politician’s ability. More complex tasks increase the probability that the type of the decision maker is revealed, and, therefore make the job more attractive for able candidates and the less for unable ones. Each citizen may run for office at either the local or the central level, or not enter the political market, in which case she earns an exogenous market wage. We show that pooling and separating equilibria at both government levels are possible, depending on the number of tasks assigned to each level.

Keywords: Endogenous candidates, decentralization, political economy

JEL Classification:
1 Introduction

The allocation of competencies across government levels varies a lot worldwide (see, e.g., Ter-Minassian, 1997, OECD 1999, 2002, Stegarescu 2005, Arzaghi and Henderson, 2005). The fiscal federalism literature, starting with Oates (1972) seminal work, provides different explanations for these differences, based on whether (or de degree to which) the public good is subject to scale economies, it generates inter-regional spillovers, or it must target heterogeneous local preferences. More recently, the political economy literature has joined the debate, putting forward a powerful argument in favor of decentralization, namely, that it provides voters with a discipline mechanism based on yardstick competition. With correlated economic contexts, voters may use the performance of neighbor jurisdictions to condition the reelection of the official in their own (Belleflamme and Hindriks, 2005, Hindriks and Lockwood, 2009).

This paper sheds light on the previously unnoticed effect of the assignment of tasks between the local and central government on the quality of politicians. Indeed, good policy outcomes result from an institutional context that provides the right incentives either to implement good policies (e.g., yardstick competition), or to foster the entry of good politicians into the political market. Both have been documented empirically (see, e.g., Besley and Case on yardstick competition and Besley et al., 2005 on candidate selection). Starting with Besley and Coate (1997) and Osborne and Slivinski (1996) citizen-candidate models, the literature has developed several arguments explaining the quality of politicians. Caselli and Morelli (2004) offer an explanation for why incompetent individuals have a comparative advantage in running for political office, based on their lower outside option, while Poutvarra and Takalo (2005) show that the rewards paid to politicians are successful in selecting good candidates only if campaign costs are sufficiently high.

This paper is the first to propose a theory of candidate quality based on the coexistence of two government layers. We argue that the assignment of tasks to the different government levels has an impact on the quality of the pool of agents that run for each level. In a nutshell, our model shows that good politicians are attracted by complex political jobs. We setup a model where a political job is composed of several tasks, and the outcome of each task is a random variable with a higher expected value, the more competent is the politician that performs it. Voters observe the outcome of the different tasks but not the politician’s ability. In this framework, the more complex the tasks assigned to a particular level of government, the higher the probability that the type of the decision maker is revealed, and, therefore the more attractive that level of government is for able candidates and the less it is to unable ones. Our citizen-candidate model allows agents to run for office at either the local or the central level, or not to enter the political market, in which case they earn an exogenous market wage. We characterize the quality of the polity at both the central and local level of government depending on the number of tasks assigned to each level, and assuming that the local government never handles more tasks than the central one. Our main results are as follows. Firstly, low quality candidates never run at the central level; secondly, high quality candidates run at the central level, and may also run at the local level if its complexity is high enough; thirdly, increasing the complexity of the political job at the local level increases the proportion of high quality local candidates.

(What follows has still to be shown)
Our analysis reveals a trade-off in the assignment of tasks between the two government levels. Indeed, assigning more tasks to the local level increases the proportion of high quality local politicians, but it may decrease the expected quality of the performed tasks because, it decreases the number of central tasks, which are the only ones that will be performed by a good politician for sure. This induces a non-monotonic relationship between average task outcome and the number of tasks assigned to the local level. Full centralization (i.e., no tasks performed by the local level) is the most efficient organization; but once one starts to allocate some tasks to the local level, then efficiency decreases at first, and then starts to increase once the number of tasks is sufficiently high so as to attract good politicians for the local government.

A full-fledged efficiency analysis must take into account the costs incurred by the candidates. In our framework, this costs are important as they serve as a selection device.

The paper is organized as follows. In Section 2 we present the model with only one level of government and show how the complexity of the task to be performed has an impact on the expected quality of candidates. Section 3 presents the two-tier government setup and derives our main results.

2 The Model

The economy is populated by overlapping generations of $2c + 1$ agents, who live for two periods and do not discount the future. There are two types of agents, the competent (or good) and the incompetent (or bad), denoted $\theta = g, b$, in equal proportions; hence, each individual agent perceives the remaining population as including $c$ individuals of each type. In each period, the agents must decide whether to enter the political market, that is, become politicians; all the agents who either do not enter the political market or do enter, but are not elected enjoy a wage in the private market, denoted $w_b < w_g < \mu$ which is increasing in their competence level and lower than the ego rent.\textsuperscript{1} We assume that the value a good agent working “on the market” increases with seniority, implying that a good old agent will never run for office. Hence, an old candidate signals herself as bad and is never elected. All the politicians running for office are, thus young, and the voters do not get any signal about their quality. Therefore, they all face an equal chance of winning the election, which we denote $q$. The agents derive an ego-rent of $\mu$ from holding the political office, and pay an idiosyncratic entry cost $\gamma$, assumed to follow a uniform distribution between 0 and $\bar{\gamma}$. The entry process generates an endogenous probability that any given politician is of the good type, which we call the quality of the polity and denote $\pi$.\textsuperscript{2}

The political office comprises $n$ tasks, a higher number of tasks implying a greater complexity of the office. The politician in charge is responsible for all the tasks. The outcome of each task is a normal random variable with variance $\sigma^2$ and expectation $\lambda_\theta$ with $\lambda_g > \lambda_b$. We are thus assuming that politicians’ competence changes the expected outcome of a task, but not its variability. Voters observe a vector of task outcomes $x = (x_1, x_2, \ldots, x_n)$. Given the normality assumption, the probability that a type $\theta$

\textsuperscript{1} This assumption ensures that the set of politicians is non-empty.

\textsuperscript{2} We show below that $\pi$ is time-invariant.
politician generates vector $\mathbf{x}$ is

$$v(\mathbf{x}, \lambda_g) = \prod_{j=1}^n \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_j-\lambda_g)^2}{2\sigma^2}} = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{\sum_{j=1}^n (x_j-\lambda_g)^2}{2\sigma^2}}$$

Politicians are term-limited, and can only be re-elected once. At the end of the first period in office, the voters compute the posterior probability that the politician is good, given the observed performance. If the posterior probability is lower than the prior $\pi$, then the incumbent is ousted from office and a new politician is randomly selected from the current pool of candidates. If it is higher than $\pi$, then the incumbent is ousted with an exogenous probability of $\alpha$, which may result from randomness in the voters’ decision process, possibly stemming from coordination failures, or personal reasons that lead a successful politician to an early retirement from politics.

### 2.1 The re-election stage

Voters compute the posterior probability that the politician is of the good type,

$$p(g|\mathbf{x}) = \frac{\pi v(\mathbf{x}, \lambda_g)}{\pi v(\mathbf{x}, \lambda_g) + (1-\pi)v(\mathbf{x}, \lambda_b)}$$

However, voters make mistakes and even when the posterior probability is greater than the prior $(\pi)$, they may oust the politician with a probability of $\alpha$. For $\pi \in (0,1)$, the updated probability of facing a good politician is greater than the prior if and only if

$$v(\mathbf{x}, \lambda_g) > v(\mathbf{x}, \lambda_b)$$

which, after straightforward simplification, becomes

$$\frac{\sum_{j=1}^n x_j}{n} > \frac{\lambda_g + \lambda_b}{2}$$

(1)

Using (1) and recalling that the distribution of the sample average follows a normal distribution with expectation $\lambda_p$ and variance $\sigma^2/n$ the properties of the normal distribution, the probability that a good politician is re-elected is given by $P_g - \alpha$, where

$$P_g = P\left(\frac{\sum_{j=1}^n x_j}{n} > \frac{\lambda_g + \lambda_b}{2}\right) = 1 - P\left(\frac{\sum_{j=1}^n x_j/n - \lambda_g}{\sqrt{\sigma^2/n}} \leq \frac{(\lambda_g + \lambda_b)/2 - \lambda_g}{\sqrt{\sigma^2/n}}\right)$$

$$= 1 - \Phi\left(\frac{\sqrt{n} \lambda_b - \lambda_g}{\sigma} \frac{2}{2}\right)$$

(2)

Where $\Phi(\cdot)$ is the distribution function of the standardized normal distribution. Analogously, the probability that a bad politician is re-elected is $P_b - \alpha$, with

$$P_b = 1 - \Phi\left(\frac{\sqrt{n} \lambda_g - \lambda_b}{\sigma} \frac{2}{2}\right)$$

(3)

A few interesting properties are apparent from (2) and (3). Firstly, using the symmetry of the normal distribution,

$$P_g = 1 - P_b$$

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Secondly, the fact that $\lambda_g > \lambda_b$ ensures that the good politician faces a higher probability of being re-elected than not, while the reverse happens for the bad one, i.e.

$$P_g > \frac{1}{2} > P_b$$

Thirdly, increasing the complexity of the political task increases the re-election chances of the good politician, while it decreases those of the bad one. Indeed,

$$\frac{\partial P_g}{\partial n} = -\frac{\lambda_b - \lambda_g}{2\sigma^2} \phi \left( \frac{\sqrt{n} (\lambda_b - \lambda_g)}{2} \right) > 0$$ (4)

$$\frac{\partial P_b}{\partial n} = -\frac{\lambda_g - \lambda_b}{2\sigma^2} \phi \left( \frac{\sqrt{n} (\lambda_g - \lambda_b)}{2} \right) = -\frac{\partial P_g}{\partial n} < 0$$ (5)

where $\phi(\cdot)$ stands for the standardized normal density. This property is very important for our results, for it ensures that more complex political jobs (i.e., those with higher number of tasks) are relatively more attractive for good politicians.

### 2.2 The entry decision

The re-election probabilities, derived below, depend on the agent’s type, and are denoted $\rho_\theta = P_\theta (1 - \alpha)$, $\theta = b, g$, where $\alpha \in (0, 1)$. Recall that holding office entails an ego-rent of $\mu$, the outside option is $w_g > w_b$, and the idiosyncratic costs to run for office, denoted $\gamma$ are drawn from a uniform distribution between 0 and $\bar{\gamma}$. The expected utility of running for office for a candidate of type $\theta = g, b$ is thus given by

$$q(1 + \rho_\theta)\mu + (1 - \rho_\theta)w_\theta + (1 - q)2w_\theta - \gamma$$

which simplifies to

$$q(\mu - w_\theta)(1 + \rho_\theta) + 2w_\theta - \gamma$$

This expected utility, when compared to the value of not running for office, $2w_\theta$, $\theta = b, g$ yields a cut-off entry cost of

$$\hat{\gamma}_\theta = (\mu - w_\theta)q(1 + \rho_\theta), \theta = g, b$$ (6)

which is increasing in both the election and re-election probabilities, i.e., the agents are willing to pay a higher cost to enter the political market if they face better election prospects.

The quality of the polity, defined as the probability that any given candidate is of good quality, is given by

$$\pi = \frac{\hat{\gamma}_g}{\hat{\gamma}_g + \hat{\gamma}_b}$$

We now show that the quality of the polity is stationary. To see why, let us now look at the entry decision of the young agents in any period after the initial one. An individual agent observes the performance of the politician who held office in the first period, and two situations may arise. Firstly, her performance may be bad, in which case she is voted out of office, and the decision is the same as in the first period. Now suppose that her performance is good. In that case, there is a probability of $\alpha$ that she is voted out of office. Crucially, the agent makes her entry decision after performance is revealed, but
before the uncertainty about ousting an incumbent with a good performance is resolved. The expected value of entering the political market is then

$$(1 - \alpha)2w_\theta + \alpha (\tilde{q}((1 + \rho_\theta)\mu + (1 - \rho_\theta)w_\theta) + (1 - \tilde{q})w_\theta) - \gamma$$

where $\tilde{q}$ denotes the election probability in periods after the initial one, which, compared to the value of not entering the political market, yields a cut-off entry cost of

$$\tilde{\gamma}_\theta = \alpha \tilde{q}(1 + \rho_\theta)(\mu - w_b)$$

implying that the quality of the polity is stationary.

From (6), the total number of agents in the political market is

$$c \hat{\gamma}_g + \hat{\gamma}_b$$

Hence, the election probability for an individual agent is given by

$$q = \frac{\tilde{\gamma}}{\hat{\gamma} + c(\hat{\gamma}_g + \hat{\gamma}_b)} \quad (7)$$

Armed with these preliminary results, we derive the equilibrium of the entry game in the next subsection, and show the fundamental relationship between complexity and average candidate quality.

2.3 Complexity and candidate quality

The number and average quality of candidates on the market is the result of the system of equations formed by

$$\hat{\gamma}_g + \hat{\gamma}_b = (\mu - w_g)q(1 + \rho_g) + (\mu - w_b)q(1 + \rho_b) \quad (8)$$

and (7) has a fixed point provided that

$$(\mu - w_b)(1 + \rho_b) + (\mu - w_g)(1 + \rho_g) \leq 2\tilde{\gamma}(1 + 2c)$$

Noting that the right hand side is increasing in $P_b$, ad that $0 < P_b < 1/2$, a sufficient condition for the above inequality is

Assumption 1 $(2\mu - w_b - w_g)(3 - 2\alpha) \leq 4\tilde{\gamma}(1 + 2c)$

which we assume hereafter. One may now compute the impact of increased complexity on the number of candidates on the marker and the election probability. Notice that (7) does not change with $n$, neither does the intersection of (8), while its slope with respect to $q$ decreases:

$$\frac{\partial P_g}{\partial n}(w_b - w_g) < 0$$

$^3$Would this not be fulfilled, we would have a corner solution for least one of the type politicans i.e. all potential candidates of that type would enter the political market.

$^4$The fixed point is the intersection between a linear increasing function, the inverse of (8), with intersections at $(0, 0)$ and $(\mu - w_b)(1 + P_b - \alpha) + (\mu - w_g)(1 + P_g - \alpha), 1)$ and a downward sloping convex curve $q(\hat{\gamma}_b + \hat{\gamma}_g)$, given by (7) intersections at $(0, 1)$ and $(2\tilde{\gamma}, (1 + 2c)^{-1})$. 

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Hence, following an increase in the task complexity, the number of candidates on the political market decreases, i.e., $\hat{\gamma}_g^* + \hat{\gamma}_b^*$ decreases while the election probability $q_g^*$ increases, where $*$ is used to denote equilibrium variables.

It is also straightforward to show that

$$\frac{\partial \hat{\gamma}_g^*}{\partial n} = (\mu - w_g) \left( (1 + P_g) \frac{\partial q_g^*}{\partial n} + q_g^* \frac{\partial P_g}{\partial n} \right) > 0$$

From which it follows that $\hat{\gamma}_b$ must decrease.

We summarize our results in the following proposition.

**Proposition 1** When the complexity of the political task increases, the number of good politicians on the market increases, while that of the bad ones decreases. Overall, there are less politicians on the political market.

Our setup has the interesting property that it endogenously generates a better polity when the political job is more complex. We now extend it to include two government layers, with the aim of characterizing how the assignment of tasks to the local government drives the average quality of candidates running both for the central and local governments.

### 3 The equilibrium with two government levels

We now suppose that there are two government levels, the central and the local one, with complexity $n^c$ and $n^l < n^c$, respectively, implying that competent agents face better re-election prospects at the central level, while incompetent ones face better re-election prospects at the local level. We further suppose that there is a large number of tasks to allocate between the two levels, so that $n^c \in [n/2, n]$ and $n^l \in (0, n/2]$, with $n^c + n^l = n$. Notice that at the lower bound of $n^c$, we have $P_{bg}^c = 1 - P_{bg}^c < P_{bg}^l = P_{bg}^c = \bar{p}$, while at the upper bound $P_{bg}^c = P_{bg}^l = 1/2$, $P_{bg}^c = \bar{p}$. Importantly, we assume that both government levels must exist, so $n_l$ is bounded away from zero.

The ego-rent generated by the central office is normalised to $\mu^c = 1$, without loss of generality, while that of the local office is lower, and given by $\mu^l = \mu \leq 1$. The outside options are normalised such that $w_g = w_b = 0$. In each period, each individual agent decides whether to enter the political market at the central or local level, or not to enter at all. The expected utility of a type $\theta = g, b$ citizen entering at level $i = c, l$ is then

$$U_i^\theta = q^i \mu^i (1 + \rho_b^i) - \gamma$$

An agent becomes a politician at the central level if $U_c^g \geq U_c^b \geq 0$, at the local level if $U_l^b \geq U_l^c \geq 0$, and does not enter the political market otherwise. In what follows, we shall use $\hat{\gamma}_\theta^i$ to denote the marginal entry cost of a type $\theta$ agent entering the political market at level $i$.

There are several possible equilibria, depending on whether each type of politician operates at the central, local or both government levels. However, the fact that the central government is more complex than the local one makes it a natural pole of attraction to the competent candidates, and we may state a first result.

**Proposition 2** There are always some good politicians at the central level.
Hence, there is always a positive probability that central tasks are undertaken by a good politician. The relative attractiveness of the central level has two components. The first, the ego-rent, is not type specific. The second is related to the re-election probabilities, and it is type-specific. Good types prefer the central level, while bad types prefer the local level. This discrepancy is softened as more tasks are devoluted to the local government. The interplay between these two components generates the equilibrium configuration displayed in Figure 1.

We summarise our findings in the following Propositions.

**Proposition 3** The equilibrium of the game depends on the relative attractiveness of the local government. When $\mu$ is

(i) high, the unique equilibrium is characterised by pooling of bad and good candidates at the local level, and only good candidates at the central level.

(ii) intermediate, the equilibrium depends on the devolution of tasks to the local government; less devolution generates a separating equilibrium with good candidates at the central level and bad candidates at the local level.

(iii) low, the unique equilibrium is characterised by pooling of bad and good candidates at the central level, and only bad candidates at the local level.

When the local level is very unattractive for the bad candidates, due to the combination of high task devolution and low $\mu$, one may end up with the very unpleasant situation of having no candidates, whatsoever, running at the local level. This happens despite the fact that the election probability is equal to 1 in such cases. However, the local office has a very low return and bad candidates prefer to go to the central level where they have some positive probability of earning a much higher rent. This can never happen at the central level, however. Should all the candidates cluster at the local level, a good candidate running at the central level wins in all respects: she is elected for sure, earns a higher rent, and faces better re-election prospects. We state our next result.

**Proposition 4** The local office is not filled by any politician when $\mu$ is very low and task devolution is sufficiently high.
As it is clear from Figure 1, and Propositions ?? and ??, the effect of complexity is different, depending on the range of $\mu$. It need not be the case that increasing the complexity of the local government increases the likelihood that a good politician holds it. This is the object of our next Proposition.

**Proposition 5** When $\mu$ is high, task devolution increases the proportion of good politicians at the local level. When $\mu$ is low, task devolution increases the proportion of bad politicians at the central level, eventually driving the number of politicians at the local level to zero.

This result implies that it is not necessarily desirable to increase the complexity of the local government. When the local ego-rent, or wage, is very low, good politicians are never attracted by this government level, no matter its complexity. Hence, increasing the number of tasks allocated to the local government, instead of attracting some good politicians, makes some bad politicians move to the central government, where even if they face lower re-election prospects, they enjoy a higher ego-rent. In this case, the local government is still in the hands of a bad politician for sure, and the central government now has some probability of also being held by a bad politician. Moreover, one is moving tasks which are performed by a good politician with some probability to the local government, where they are performed by a bad politician for sure. To much devolution may have an even stronger negative consequence: leaving local tasks without any official in charge, and thus not undertaken at all.

Conversely, when the local government pays sufficiently, then increasing its complexity attracts good politicians and increases the probability that both levels of government are held by a good candidate. In this case, there is a clear trade-off. Each task that is moved from the central to the local level is no longer performed for sure by a good politician; however, it increases the prospects that all the local tasks are performed by good politicians. This suggests that there is an optimal task assignment that maximizes the combined payoff from central and locally performed tasks.

One may envisage changing $\mu$, interpreted as the (relative) local political wage instead of a pure ego-rent, instead of the task assignment, as a means to influence the quality of the polity at the different government layers. This is the object of our next proposition.

**Proposition 6** Increasing the local political wage when it is

(i) very low, attracts some bad candidates to the otherwise unoccupied local government;

(ii) low, drives down the share of bad politicians at the central level to zero;

(iii) intermediate, has no impact on the equilibrium;

(iv) high, attracts an increasing share of good politicians to the central level;

This proposition implies that there is a whole range of $\mu$ for which increasing it plays no role in the quality of the polity in both government layers, hence it is a pure transfer from the tax payers to the politicians. Contrary to task devolution, the political wage, when it does play a role, it is a positive one.

These two results set the ground for the welfare analysis which we undertake in the next section.
4 Welfare analysis

5 Appendix

We begin by introducing some notation which is used in the subsequent proofs. Let

\[ x_c^c(n^c) = 1 + \rho_c^c(n^c) \]
\[ x_b^c(n^c) = \mu(1 + \rho_b^c(n^c)) \]
\[ x_c^l(n^c) = 1 + \rho_c^l(n^c) \]
\[ x_b^l(n^c) = \mu(1 + \rho_b^l(n^c)) \]

Before analyzing further the different possible configurations, we present some useful properties of \( x_j^\theta \), \( \theta = g, b, j = l, c \).

Claim 1

\[ \frac{x_c^c(n^c)}{x_b^c(n^c)} \leq \frac{x_g^c}{x_g^l} \] (10)

Where the equality arises for \( n^c = n^l = n/2 \).

Proof. It follows from the facts that \( P_c^g(n^c) \geq P_l^g(n^c) \), \( \forall n^c \), while \( P_c^b(n^c) \leq P_l^b(n^c) \), \( \forall n^c \), and that \( P_g^c(n/2) = P_g^l(n/2) \), \( \theta = c, l \). \( \blacksquare \)

Claim 2

\[ x_g^c \geq x_g^l \] (11)

Proof. It follows from \( \mu \leq 1 \) and \( P_g^c(n^c) \geq P_g^l(n^c) \), \( \forall n^c \). \( \blacksquare \)

Claim 3

\[ \frac{x_l^b}{x_b^b} \in \left[ \mu, \frac{3 - \alpha}{2} \mu \right] \]

Moreover, when \( n_c \) is low enough, \( \frac{x_l^b}{x_b^b} < 1 \).

Proof. \( \frac{x_l^b}{x_b^b} \) is increasing in \( n^c \). Moreover, \( \rho_b^l(n/2) = \rho_b^l(n/2) \), and \( \lim_{n \to \infty} \rho_b^l(n) = \frac{1 - \alpha}{2} \) and \( \rho_b^l(0) = 0 \). The Claim then follows. \( \blacksquare \)

5.1 Proof of Proposition 2

Proof. Suppose that all the good candidates run at the local level. Then, it must be the case that \( U_g^l > U_g^c \). Two cases may arise:

(i) All the bad agents who become politicians also enter at the local level. Then, anyone entering at the central level is elected for sure (\( q^c = 1 \)), and it is straightforward to use (9) to show that good politicians prefer the central level, since

\[ x_g^c > x_g^l \]

for all \( q^l \leq 1 \), recalling that \( \mu^c > \mu^l \) and \( P_g^c > P_g^l \).
(ii) There are some bad agents who become politicians at the central level. Then, the entry conditions read
\[
q^c x_g^c \geq x_b^l q^l
\]
\[
q^l x_g^l > q^c x_g^c
\]
the two inequalities can be rewritten as
\[
\frac{x_g^c}{x_g^l} < \frac{q^l}{q^c} \leq \frac{x_b^l}{x_b^c}
\]
which is impossible by claim 1.

\[\square\]

5.2 The separating case

In the separating case, there are only good politicians at the central level, and only bad ones at the local level. The cut-off entry costs are thus given by
\[
\hat{\gamma}_g = q^c x_g^c
\]
\[
\hat{\gamma}_b = q^l x_b^l
\]
Given (12), there are \(c \hat{\gamma}_c\) candidates at the central level and \(c \hat{\gamma}_b\) at the local level, hence
\[
q^i = \frac{\hat{\gamma}}{\hat{\gamma} + c \hat{\gamma}_g},
\]
for the two pairs \((i = c, \theta = g)\) and \((i = l, \theta = b)\). Using (12), together with (13), \(\hat{q}^l(n^c, \mu)\) and \(\hat{q}^c(n^c, \mu)\) solve the equations \(f^l(q^l; n^c, \mu) = 0\) and \(f^c(q^c; n^c, \mu) = 0\), given by
\[
f^l(q^l; n^c, \mu) = q^l \hat{\gamma} + c x_b^l (q^l)^2 - \hat{\gamma}
\]
\[
f^c(q^c; n^c, \mu) = q^c \hat{\gamma} + c x_g^c (q^c)^2 - \hat{\gamma}
\]
For future reference, note that \(\hat{q}^j, j = l, c\) may equivalently be implicitly given by
\[
\hat{q}^c x_g^c = \frac{\hat{\gamma}}{c} \frac{1 - \hat{q}^c}{\hat{q}^c}
\]
\[
\hat{q}^l x_b^l = \frac{\hat{\gamma} - \hat{q}^l}{c} \frac{1 - \hat{q}^l}{\hat{q}^l}
\]
We start by highlighting a few important properties of \(\hat{q}^l(n^c, \mu)\) and \(\hat{q}^c(n^c, \mu)\).

\(i)\) The continuity of \(f^j(q^j; n^c, \mu), j = l, c,\) and the fact that \(f^l(0; n^c, \mu) < 0, j = l, c,\) and \(f^l(1; n^c, \mu) > 0, j = l, c\) ensure that there exist a \(\hat{q}^l(n^c, \mu) \in (0, 1)\) and \(\hat{q}^c(n^c, \mu) \in (0, 1)\) which solve (14).
(ii) $\hat{q}^l(n^c, \mu) > \hat{q}^c(n^c, \mu)$. To see why, note that $f^l(\hat{q}^c; n^c, \mu) = \hat{q}^c \gamma + cx^{l}_{b}(\hat{q}^c)^2 - \gamma < f^c(\hat{q}^c; n^c, \mu) = 0$, where the inequality follows from $x^{c}_{b}(n^c) > x^l_{b}(n^c)$. This, together with the fact that both $f^l(q^c; n^c, \mu)$ and $f^c(q^c; n^c, \mu)$ are increasing, implies that $\hat{q}^l(n^c, \mu) > \hat{q}^c(n^c, \mu)$.

(iii) Since $\rho^c_b(n^c)$ and $\rho^c_q(n^c)$ are increasing, both $\hat{q}^c(n^c, \mu)$ and $\hat{q}^l(n^c, \mu)$ are decreasing in $n^c$:

$$\frac{d\hat{q}^l}{dn^c} = \frac{\partial \hat{q}^l}{\partial n^c} = \frac{(q^l)^2 c q^l}{\gamma + 2q^l x^l_{b} c} < 0,$$

for $(j = c, \theta = g)$, and $(j = l, \theta = b)$

$$(16)$$

(iv) $\hat{q}^l(n^c, \mu)$ is decreasing in $\mu$:

$$\frac{d\hat{q}^l}{d\mu} = -\frac{\partial \hat{q}^l}{\partial \mu} = \frac{(q^l)^2 c q^l}{\gamma + 2q^l x^l_{b} c} < 0$$

and when $\mu = 0$, $\hat{q}^l = 1$.

(v) Finally, for the pairs $(\theta = g, j = c)$, $(\theta = b, j = l)$

$$\frac{d\hat{q}^l}{dy} = \frac{\partial \hat{q}^l}{\partial y} x^l_{b} + \frac{\partial x^l_{b}}{\partial y} \hat{q}^l = \frac{\partial x^l_{b}}{\partial y} \hat{q}^l \left(1 - \frac{q^l x^l_{b}}{\gamma + 2q^l x^l_{b} c}\right) > 0, y = n^c, \mu$$

$$(17)$$

A separating equilibrium exists if and only if

$$h_b(n^c, \mu) = \hat{q}^l(n^c, \mu) x^l_{b}(n^c, \mu) - \hat{q}^c(n^c, \mu) x^c_{b}(n^c) > 0$$

$$(18)$$

$$h_g(n^c, \mu) = \hat{q}^c(n^c, \mu) x^c_{g}(n^c) - \hat{q}^l(n^c, \mu) x^l_{g}(n^c, \mu) > 0$$

$$(19)$$

We establish the following two Lemmas.

**Lemma 1** There exist $(\tilde{\mu}_1, \tilde{\mu}_2) \in (0, 1)$ such that $h_b(n^c, \mu) < 0$ for $\mu \leq \tilde{\mu}_1$, and $h_b(n^c, \mu) > 0$ for $\mu \geq \tilde{\mu}_2$. When $\tilde{\mu}_1 < \mu < \tilde{\mu}_2$, there exists $n^c_1 \in (n/2, n)$ such that $h_b(n^c, \mu) > 0$ if and only if $n^c > n^c_1$.

**Proof.**

(i) We know, by (17), that $\hat{q}^c x^c_{b}$ is increasing in $n^c$. Also, since $x^c_{b}(n^c)$ is decreasing in $n^c$, $\hat{q}^c x^c_{b}$ is decreasing in $n^c$. It follows that $\frac{dh_b}{dn^c} > 0$.

(ii) It is also easy to establish that $\frac{dh_b}{d\mu} > 0$. Indeed, $\hat{q}^c x^c_{b}$ does not change with $\mu$, while, by (17), $\hat{q}^c x^c_{b}$ increases in $\mu$. Also,

$$h_b(n^c, 0) = -\hat{q}^c(n^c, \mu) x^c_{b}(n^c) < 0, \quad h_b(n^c, 1) = \hat{q}^l(1 + \rho^c_b) - \hat{q}^c(1 + \rho^c_b) > 0,$$

given that $\hat{q}^l > \hat{q}^c$, and that $\rho^c_b > \rho^c_q$.

This, together with the monotonicity of $\hat{q}^c x^c_{b}$, implies that there exists a $\tilde{\mu}(n^c) \in (0, 1)$ such that (18) is verified if and only if $\mu > \tilde{\mu}$. 

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(iii) Finally, using (i) and (ii),

$$\frac{d\tilde{\mu}}{dn^c} = -\frac{dh_g}{dn^c} \frac{dn^c}{d\mu} < 0$$

Now set $\tilde{\mu}_1 = \tilde{\mu}(n)$, and $\tilde{\mu}_2 = \tilde{\mu}(n/2)$ and the claim follows. By (20), $\tilde{\mu}_1 > 0$, and $\tilde{\mu}_2 < 1$.

\[\square\]

**Lemma 2** There exist $(\hat{\mu}_1, \hat{\mu}_2) \in (0, 1)$ such that $h_g(n^c, \mu) > 0$ for $\mu \leq \hat{\mu}_1$, and $h_g(n^c, \mu) < 0$ for $\mu \geq \hat{\mu}_2$. When $\hat{\mu}_1 < \mu < \hat{\mu}_2$, there exists $\hat{n}_2^c \in (n/2, n)$ such that $h_g(n^c, \mu) > 0$ if and only if $n^c > \hat{n}_2^c$.

**Proof.**

(i) We know, by (17), that $\hat{q}^c x_g$ is increasing in $n^c$. Also, since $x_g^l(n^c)$ is decreasing in $n^c$, $\hat{q}^l x_g^l$ is decreasing in $n^c$. It follows that $\frac{dh_g}{dn^c} > 0$.

(ii) It is also easy to establish that $\frac{dh_g}{d\mu} < 0$. Indeed, $\hat{q}^c x_g^c$ does not change with $\mu$, while

$$\frac{d\hat{q}^l x_g^l}{d\mu} = \frac{dq^l}{d\mu} x_g^l + \frac{dx_g^l}{d\mu} \hat{q}^l - \frac{(q^l)^2 \hat{q}^l}{\gamma + 2q^l x_r^c x_g^l} x_g^l \frac{dx_g^l}{d\mu} \hat{q}^l = \frac{dx_g^l}{d\mu} \hat{q}^l \left( 1 - \frac{1 + \rho^l_g}{1 + \rho_g^l} q_g^l \right) > 0,$$

Also,

$$h_g(n^c, 0) = \hat{q}^c x_g^c > 0, \quad \text{and} \quad h_g(n^c, 1) = \hat{q}^c (1 + \rho^c_g) - \hat{q}^l (1 + \rho^l_g) < 0,$$

(21)

where the last inequality is obtained using $\rho_g^l > \rho_g^c$ and (15) to write

$$\hat{q}^l (1 + \rho^l_g) = \frac{\gamma}{c} \frac{1 - \hat{q}^l}{\hat{q}^c} \left( 1 + \rho^c_g \right) \frac{1 - \hat{q}^c}{\hat{q}^l} > \frac{\gamma}{c} \frac{1 - \hat{q}^c}{\hat{q}^l} = \hat{q}^c (1 + \rho^c_g).$$

This, together with the monotonicity of $\hat{q}^l x_g^l$, implies that there exists a $\hat{\mu}(n^c) \in (0, 1)$ such that (19) is verified if and only if $\mu < \hat{\mu}(n^c)$.

(iii) Finally, using (i) and (ii),

$$\frac{d\tilde{\mu}}{dn^c} = -\frac{dh_g}{dn^c} \frac{dn^c}{d\mu} > 0$$

Now set $\tilde{\mu}_1 = \tilde{\mu}(n/2)$, and $\tilde{\mu}_2 = \tilde{\mu}(n)$, and the claim follows. By (21), $\tilde{\mu}_1 > 0$, and $\tilde{\mu}_2 < 1$.

\[\square\]

Having analyzed (18) and (19) separately, we relate the two in the following Lemma.

**Lemma 3**

$$\tilde{\mu}_1 < \tilde{\mu}_2 = \hat{\mu}_1 < \hat{\mu}_2,$$

from which
• when \( \mu < \tilde{\mu}_1 \), (18) is not respected, hence a separating equilibrium fails to exist
• when \( \mu > \tilde{\mu}_2 \), (19) is not respected, hence a separating equilibrium fails to exist
• when \( \tilde{\mu}_1 < \mu < \tilde{\mu}_1 \), a separating equilibrium exists if and only if \( n^c > \tilde{n}_1^c \)
• when \( \tilde{\mu}_1 < \mu < \tilde{\mu}_2 \), a separating equilibrium exists if and only if \( n^c > \tilde{n}_2^c \)

**Proof.** Rewrite (18) and (19) as

\[
\begin{align*}
\hat{h}_b(n^c, \mu) &= q^c x_g^c \frac{1 + \mu_f^c}{1 + \mu_g^c} - q^l x_b^l < 0 \\
\hat{h}_g(n^c, \mu) &= q^c x_g^c - q^l x_b^l \frac{1 + \mu_f^l}{1 + \mu_g^l} > 0
\end{align*}
\]

When \( n^c = n/2, \rho_g = \rho_b, \theta = g, b \), and \( \hat{h}_b(n^c, \mu) = h_g(n^c, \mu) \), implying that \( \tilde{\mu}_2 = \hat{\mu}(n/2) = \hat{\mu}_1 = \hat{\mu}(n/2) \). It then follows that \( \hat{\mu}_1 < \tilde{\mu}_2 = \hat{\mu}_1 < \hat{\mu}_2 \).

**5.3 The equilibrium with pooling at the central level**

The cut-off entry costs in this case are

\[
\begin{align*}
\hat{\gamma}_g &= q^c x_g^c \\
\hat{\gamma}_b &= q^l x_b^l = q^c x_b^c
\end{align*}
\]

Let \( \theta \) be the endogenous share of the bad politicians going at the central level. In that case, there are \( c \frac{\gamma + \theta}{\gamma} \) candidates at the central level and \( c \frac{(1-\theta)}{\gamma} \) at the local level, and election probabilities are implicitly given by

\[
\begin{align*}
\hat{f}^l(q^l; n^c, \mu, \theta) &= q^l \hat{\gamma} + cx_b^l (1 - \theta) \left( q^l \right)^2 - \hat{\gamma} \\
\hat{f}^c(q^c; n^c, \mu, \theta) &= q^c \hat{\gamma} + cx_g^c (q^c)^2 + c d q^c q^l x_b^l - \hat{\gamma}
\end{align*}
\]

We start by highlighting a few important properties of \( \hat{q}^l \) and \( \hat{q}^c \).

(i) The continuity of \( f^j(q^j; n^c, \mu, \theta), j = l, c \), and the fact that \( f^l(0; n^c, \mu, \theta) < 0, j = l, c \), and \( f^l(1; n^c, \mu, \theta) > 0, j = l, c \) ensure that there exist a \( \hat{q}^l(n^c, \mu, \theta) \in (0, 1) \) and \( \hat{q}^c(n^c, \mu, \theta) \in (0, 1) \) which solve (22).

(ii) Both \( \hat{q}^c(n^c, \mu, \theta) \) and \( \hat{q}^l(n^c, \mu, \theta) \) are decreasing in \( n^c \). Firstly, note that

\[
\begin{align*}
\frac{dq^l}{dn^c} &= \frac{\partial q^l}{\partial n^c} = - \frac{(q^l)^2}{\hat{\gamma} + 2q^l x_b^l c(1 - \theta)} = - \frac{(q^l)^2}{\hat{\gamma} + 2q^l x_b^l c(1 - \theta)} < 0
\end{align*}
\]

And now use the fact that

\[
\begin{align*}
\frac{dq^l x_b^l}{dy} &= \frac{dq^l}{dy} x_b^l + \frac{dx_b^l}{dy} q^l = \frac{dx_b^l}{dy} q^l \left( 1 - \frac{q^l x_b^l c}{\hat{\gamma} + 2q^l x_b^l c(1 - \theta)} \right) > 0, y = n^c, \mu
\end{align*}
\]
to obtain
\[
\frac{d\hat{q}^c}{dn} = -\frac{\partial f^c}{\partial n} + \frac{\partial f^c}{\partial (q^c x^b)} \frac{\partial (q^c x^b)}{dn} = \frac{(q^c)^2 c d\hat{q}^c}{\partial q^c} + c\theta q^c \frac{\partial (q^c x^b)}{dn} = \frac{- (q^c)^2 c}{\hat{\gamma} + 2q^c x^c_{\mu} c} < 0
\]

(iii) Both \(\hat{q}^l(n^c, \mu)\) and \(\hat{c}^c(n^c, \mu)\) are decreasing in \(\mu\): Firstly, note that
\[
\frac{d\hat{q}^l}{d\mu} = -\frac{\partial f^l}{\partial \mu} - \frac{(q^c)^2 c d\hat{q}^l}{\partial q^c} = \frac{- (q^c)^2 c}{\hat{\gamma} + 2q^c x^c_{\mu} c} < 0
\]
And use \(\frac{d\hat{q}^l x^l}{d\mu} > 0\), to obtain
\[
\frac{d\hat{c}^c}{d\mu} = -\frac{\partial f^c}{\partial \mu} + \frac{\partial f^c}{\partial (q^c x^b)} \frac{\partial (q^c x^b)}{d\mu} = \frac{(q^c)^2 c d\hat{c}^c}{\partial q^c} + c\theta q^c \frac{\partial (q^c x^b)}{d\mu} = \frac{- (q^c)^2 c}{\hat{\gamma} + 2q^c x^c_{\mu} c} < 0
\]

(iv) \(\hat{q}^l(n^c, \mu, \theta)\) is increasing in \(\theta\), while \(\hat{c}^c(n^c, \mu, \theta)\) is decreasing in \(\theta\): Indeed,
\[
\frac{d\hat{q}^l}{d\theta} = -\frac{\partial f^l}{\partial \theta} = \frac{- (q^c)^2 c x^l_{\mu}}{\hat{\gamma} + 2q^c x^c_{\mu} c(1 - \theta)} > 0,
\]
while
\[
\frac{d\hat{c}^c}{d\theta} = -\frac{\partial f^c}{\partial \theta} - \frac{\partial f^c}{\partial (q^c x^b)} \frac{\partial (q^c x^b)}{d\theta} = \frac{- c q^c x^c_{\mu} + c q^c \theta x^l_{\mu} \frac{d\hat{q}^l}{d\theta}}{\hat{\gamma} + 2q^c x^c_{\mu} c} < 0
\]
Moreover, \(\hat{q}^l(n^c, \mu, 1) = 1\).

(v) Finally, \(\hat{q}^l(n^c, \mu, \theta) > \hat{c}^c(n^c, \mu, \theta)\). To see why, note that \(\hat{q}^l(n^c, \mu, 0) > \hat{c}^c(n^c, \mu, 0)\) by the properties of the separating equilibrium. Then, by (iv), \(\hat{q}^l(n^c, \mu, \theta) > \hat{c}^c(n^c, \mu, \theta) > 0, \forall \theta > 0\).

The equilibrium exists if and only if
\[
\begin{align*}
h_b(n^c, \mu, \theta) &= \hat{q}^l(n^c, \mu, \theta) x^l(n^c, \mu) - \hat{c}^c(n^c, \mu, \theta) x^c_{\mu}(n^c) = 0 \quad (23) \\
h_g(n^c, \mu, \theta) &= \hat{c}^c(n^c, \mu, \theta) x^c_{\mu}(n^c) - \hat{q}^l(n^c, \mu, \theta) x^l(n^c, \mu) > 0 \quad (24)
\end{align*}
\]

We establish the following two Lemmas.

**Lemma 4** Whenever \(h_b(n^c, \mu, \theta) = 0, \ h_g(n^c, \mu, \theta) > 0\).

**Proof.** Rewrite
\[
h_b(n^c, \mu, \theta) = \hat{q}^l(n^c, \mu, \theta) x^l(n^c, \mu) - \hat{c}^c(n^c, \mu, \theta) x^c_{\mu}(n^c)
\]
which, when \(h_b(n^c, \mu, \theta) = 0\), reduces to
\[
h_g(n^c, \mu, \theta) = \hat{c}^c(n^c, \mu, \theta) x^c_{\mu}(n^c) - \hat{q}^l(n^c, \mu, \theta) x^l(n^c, \mu) > 0,
\]
where the inequality follows from Claim 1. ■
Lemma 5 When \( \mu < \tilde{\mu}(n^c) \), there exists an equilibrium with pooling at the central level, characterised by

\[
\frac{d\hat{\theta}}{dn^c} = -\frac{\partial h_b}{\partial n^c} < 0 \\
\frac{d\hat{\theta}}{d\mu} = -\frac{\partial h_b}{\partial \mu} < 0
\]

i.e., the share of bad politicians in the central government is a decreasing function of both the central government complexity and the attractiveness of the local government.

Proof. By Lemma 4, a sufficient and necessary condition for the equilibrium to exist is that \( h_b(n^c, \mu, \theta) = 0 \). By (ii), (iii), and (v) above, \( h_b(n^c, \mu, \theta) \) is an increasing function of \( n^c, \mu, \) and \( \theta \).

From Lemma 1, we know that \( h_b(n^c, \mu, 0) > 0 \) when \( \mu > \tilde{\mu}(n^c) \). Now \( h_b(n^c, 0, 1) = -\hat{q}^c x_b^c < 0 \), and \( h_b(n^c, 1, 1) = (1 + \rho_b^c) - \hat{q}^c(1 + \rho_b^c) > 0 \) where we use the fact that \( \hat{q}^c(n^c, \mu, 1) = 1 \), by (iv) above. Since \( h_b \) is an increasing function of \( \mu \), this implies that there exists a \( \tilde{\mu}(n^c) \in (0, 1) \) such that when \( \mu > \tilde{\mu}(n^c) \), \( h_b(n^c, 1, 1) > 0 \). Moreover, given that \( h_b \) is increasing in \( \theta \), \( \tilde{\mu}(n^c) < \tilde{\mu}(n^c) \). We thus have three relevant intervals. If \( \mu > \tilde{\mu}(n^c) \), \( h_b(n^c, \mu, \theta) > 0, \forall \theta \). If \( \mu < \tilde{\mu}(n^c) \), \( h_b(n^c, \mu, \theta) < 0, \forall \theta \). When \( \tilde{\mu}(n^c) < \mu < \tilde{\mu}(n^c) \), there exists \( \hat{\theta}(\mu, n^c) \) such that \( h_b(n^c, \mu, \hat{\theta}) = 0 \). Moreover,

\[
\frac{d\hat{\theta}}{dn^c} = -\frac{\partial h_b}{\partial n^c} < 0 \\
\frac{d\hat{\theta}}{d\mu} = -\frac{\partial h_b}{\partial \mu} < 0
\]

When \( \mu < \tilde{\mu}(n^c) \), \( h_b(n^c, \mu, 1) < 0 \) for all \( \mu \) and all \( n^c \), and all the politicians run at the central level. \( \blacksquare \)

5.4 The equilibrium with pooling at the local level

The cut-off entry costs in this case are

\[
\tilde{\gamma}_g = q^c x_g^c = q^l x_g^l \\
\tilde{\gamma}_b = q^l x_b^l
\]

Let \( \theta \) be the endogenous share of the good politicians going at the central level. In that case, there are \( c(1-\hat{\gamma}) \) candidates at the central level and \( c(1-\hat{\gamma})\frac{(1-\hat{\theta})\gamma_b + \gamma_b}{\tilde{\gamma}} \) at the local level, and election probabilities are implicitly given by

\[
\begin{align*}
\hat{f}^c(q^c; n^c, \mu, \theta) &= q^c \hat{\gamma} + c x_g^c (q^c)^2 + c(1-\theta)q^c q^c x_g^c - \hat{\gamma} \\
\hat{f}^l(q^l; n^c, \mu, \theta) &= q^l \hat{\gamma} + c x_g^l (q^l)^2 - \hat{\gamma}
\end{align*}
\]  

We start by highlighting a few important properties of \( \hat{q}^l \) and \( \hat{q}^c \).
Lemma 6 Whenever \( h_g(n^c, \mu, \theta) = 0 \), \( h_b(n^c, \mu, \theta) > 0 \).

Proof. Rewrite

\[
h_b(n^c, \mu, \theta) = \dot{q}^l(n^c, \mu, \theta) x^l_g(n^c, \mu) \frac{x^l_b(n^c, \mu)}{x^l_g(n^c, \mu)} - \ddot{q}^c(n^c, \mu, \theta) x^c_g(n^c) \frac{x^c_b(n^c)}{x^c_g(n^c)}
\]

which, when \( h_g(n^c, \mu, \theta) = 0 \), reduces to

\[
h_b(n^c, \mu, \theta) = \dot{q}^l(n^c, \mu, \theta) x^l_g(n^c) \left( \frac{x^l_b(n^c, \mu)}{x^l_g(n^c, \mu)} - \frac{x^c_b(n^c)}{x^c_g(n^c)} \right) > 0,
\]

where the inequality follows from Claim 1. ■
Lemma 7 When $\mu > \tilde{\mu}_2$, or $\tilde{\mu}_1 < \mu < \tilde{\mu}_2$ and $n^c < \tilde{n}^c$, there exists a continuum of equilibria characterised by

$$
\frac{d\theta}{dn^c} = -\frac{\partial h_g}{\partial n^c} > 0
$$

$$
\frac{d\theta}{d\mu} = -\frac{\partial h_g}{\partial \mu} < 0
$$

i.e., the share of good politicians at the central level is an increasing function of central government complexity, and a decreasing function of the attractiveness of the local government.

Proof. By Lemma 6, a sufficient and necessary condition for the equilibrium to exist is that $h_g(n^c, \mu, \theta) = 0$. Using (iii) above and straightforward algebra allows one to obtain

$$
\frac{d\theta}{d\mu} = q^l x_g^l \left( 1 - \frac{q^l x_g^l c}{\gamma + 2q^l x_g^l c + (1 - \theta) cq^l x_g^l} \right) > 0
$$

which, together with (iii), and (iv) above, implies that $h_g(n^c, \mu, \theta)$ is an increasing function of $n^c$, and a decreasing function of $\mu$, and $\theta$. Now $h_g(n^c, \mu, 0) = x_g^c - \tilde{q}^l x_g^l > 0$, where we use the fact that $\tilde{q}^l(n^c, \mu, 0) = 1$, by (iv) above. From Lemma 2, we know that $h_g(n^c, \mu, 1) > 0$ when $\mu < \tilde{\mu}(n^c)$. In such cases, $h_g(n^c, \mu, \theta) > 0$, $\forall \theta$. When $\mu > \tilde{\mu}(n^c)$, there exists $\theta(\mu, n^c)$ such that $h_g(n^c, \mu, \theta) = 0$. Moreover,

$$
\frac{d\theta}{dn^c} = -\frac{\partial h_g}{\partial n^c} > 0
$$

$$
\frac{d\theta}{d\mu} = -\frac{\partial h_g}{\partial \mu} < 0
$$

Lemma 8 The expected quality of the politicians at the local level is increasing with the complexity of the local tasks

Proof. The probability to have a high quality candidate at the local level is given by

$$
\frac{(1 - \theta(n_c))\tilde{\gamma}_g c}{(1 - \theta(n_c))\tilde{\gamma}_g c + \tilde{\gamma}_c} = \frac{(1 - \theta(n_c))x_g^l}{(1 - \theta(n_c))x_g^l + x_b^l}
$$

As $\theta(n_c)$ and $x_g^l$ are increasing in $n_c$ and $x_g^l$ is decreasing in $n_c$, we have that the ratio is decreasing in $n_c$. QED.
References


