Uncertainty about the Mass of Speculators and the Decision to Abandon a Currency Peg

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Abstract

The model we propose in this paper considers a rational and forward-looking policy maker in an environment in which the mass of speculators is the unknown state of the economy. Albeit the policy maker is not capable of inferring if the mass of speculators is sufficiently large in order to trigger a successful speculative attack on the currency, the fraction of speculators who place a buy order for the foreign currency signals to the policy maker the state of the economy. The main result of the paper shows the existence of at least one equilibrium in which the policy maker opts to relinquish the currency peg if and only if the quantity of buy orders is greater than the critical mass. In this model, currency crises are caused by the endogenous coordination of speculators, regardless of fundamentals.

Keywords: Speculative attacks, currency peg, mass of speculators, forward-looking policy maker, signalling.

JEL classification: F31, D82, D84

(Preliminary version; please do not quote without permission)

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1 Introduction

The question as to whether currency crises are driven by fundamentals or extraneous self-fulfilling beliefs has been exhaustively discussed in the literature on speculative attacks. According to the fundamentalist view, currency crises are the by-product of unsustainable fiscal and monetary policies. Crises would therefore be perfectly predictable conditional on macroeconomic fundamentals. On the other hand, advocates of the self-fulfilling view argue that speculative attacks are prompted by self-fulfilling expectations. Coordination problems would lead up to financial crises driven by extraneous sunspots. As a matter of fact, the debate about the determinants of currency crises still remains unsolved.

This paper analyzes the dynamic strategic interaction between speculators and a rational and forward-looking policy maker, in an environment in which there exists uncertainty regarding the mass of potential speculators in the economy. The exogenous uncertainty in the economy accrues from the fact that the policy maker does not know whether the mass of speculators is sufficiently large in order to bring about a successful speculative attack against the currency.

It can be useful to provide a brief overview of the recent literature on financial crises and information on which this paper is based.

Resorting to global games, Morris and Shin (1998) model the strategic interaction between the government and a group of speculators in the foreign exchange market, and demonstrate the uniqueness of equilibrium when there is a small amount of idiosyncratic private information about fundamentals. They prove the existence of a unique critical state such that, in any equilibrium of the game with imperfect information, the government decides to abandon the currency peg if and only if the fundamentals fall below this unique threshold. Equilibrium multiplicity associated with self-fulfilling beliefs would be the consequence of assuming that the economic fundamentals
are common knowledge.

The modelling of incomplete information is also addressed in a dynamic model proposed by Chamley (2003), in which the exchange rate is allowed to float within a predetermined band of fluctuation.\(^1\) The uncertain parameter in the economy is a mass of speculators with relatively high beliefs concerning their capacity to induce a successful speculative attack against the currency. By observing the exchange rate within the band at the end of each period, the speculators can accordingly learn whether their mass is sufficient for causing the abandonment of the peg. Notice that the observation of the exchange rate is observationally equivalent to the total demand for foreign currency, which in turn conveys a signal about the unknown state of the economy given by the fraction of active speculators (i.e., the sum of the fraction of agents who hold the foreign currency with the fraction of agents who place a buy order for the foreign currency). The observation of the exchange rate then enables agents to update their beliefs in a Bayesian fashion, so that the subgame in a determined period depends only on the fraction of agents who hold the foreign currency at the beginning of that period, as well as the speculator’s probability of the high state. The Central Bank is just assumed to match the trade orders.

Corsetti, Dasgupta, Morris and Shin (2004) extend the incomplete information game formulation used in Morris and Shin (1998) by considering the role played by a large trader in a theoretical model of speculative attacks, in which a single large speculator and a continuum of small speculators independently decide whether to attack a currency based on their private information about fundamentals. In their model, a large trader in the market might exacerbate a crisis, insofar as his presence makes the small traders more aggressive. It should be noted that they do not model explicitly the

\(^1\) This same exchange rate arrangement, with some variations, was implemented by the Brazilian economy in the middle of 1994, taking the Real Plan starting point as 1st July. The peg was abandoned soon after the re-election of President Fernando Henrique Cardoso, in January 1999.
decision of the policy makers concerning relinquishing the peg.

Chari and Kehoe (2003) resort to the herding hypothesis to model currency attacks. They develop a model of herd behavior, in which investors receive signals about the returns to risky investment and the unknown state of the economy, and decide whether or not to invest by taking into account the previous actions of those who have chosen before them. This behavior can start stampedes of investment as well as stampedes of no investment, since agents might follow the others’ actions regardless of their own signals, and hence could serve as an explanation of the random component of capital flows that drive financial crises, as claimed by them. Furthermore, it could also help to target the question of the unpredictability of currency crises, in the sense that even conditioning on macroeconomic fundamentals, crises cannot be predicted accurately.

Broner (2003) criticizes Chari and Kehoe’s model of currency crises as a sequence of one-period games, since in this way the dynamic strategic interactions among agents are missed. Broner’s paper models currency crises as a dynamic game by means of a generalization of the Krugman-Flood-Garber model, assuming that only a fraction of informed consumers know the threshold level of reserves at Central Bank with which the abandonment of the peg is associated. The finding that currency crises can be unpredictable and associated with discrete devaluations is shown by the paper, the main contribution of which is to shed light on the question of whether crises are the result of economic policy mismanagement or self-fulfilling expectations. By introducing private information in a first generation setting, Broner’s paper helps reconcile the view of currency crises as a by-product of weak and deteriorating macroeconomic fundamentals with the existence of multiple equilibria and discrete devaluations.

Angeletos, Hellwig and Pavan (2003) consider a model of regime change where agents play a global coordination game with heterogeneous information. There exists a policy maker interested in defending the status quo,
who controls a policy instrument that affects the agents’ payoff from attacking and hence the probability of regime change. The strength of the status quo is given by the fundamentals, which is private information to the policy maker. When fundamentals are confined to a critical range, the regime is sound although vulnerable to a sufficiently large attack triggered by the mass of agents. Since raising the policy is costly, the policy maker will be prone to intervening only for an intermediate range of fundamentals, which signals to the market participants that the fundamentals are neither too weak nor too strong, then enabling them to use the signal conveyed by the policy intervention as a coordination device on an aggressive or a lenient behavior. When there is uncertainty regarding the aggressiveness of market expectations, there exist sunspot equilibria, in which the status quo is abandoned with probability less than one for fundamentals belonging to an intermediate range.

A reduced form representation of the policy maker’s decision regarding defending or not the fixed peg is presented in a stylised model by Jeanne and Masson (2000). They construct sunspot equilibria in which the economy jumps across states with different levels of devaluation expectations. A sunspot variable which coordinates the private sector expectations on one state or the other might drive the jumps between states. They also propose an econometric technique based on the Markov-switching regimes framework, and find that their model explains better the recent experience of the French franc when the devaluation expectations are influenced by sunspots.

Surti (2004) develops a model in which the liquidity held by each trader is assumed to be private information. With uncertainty regarding market liquidity, it is no longer possible to envisage the strength of the Central Bank to defend the peg. Speculators will then follow others’ actions, wishing at the same time to preempt the market. The likelihood of a crisis depends on the distribution of liquidity within the market. If there is no equilibrium with a speculative attack, then the market can infer that the likelihood of a
successful attack against the peg is low due to insufficient liquidity. In the unique equilibrium of the model, only traders with liquidity above a critical threshold are prone to engaging in speculative attacks. The Central Bank’s strategy is just a parameter of the model.

The policy maker considered in this paper is rational and forward-looking in the sense of Pastine’s (2002) paper. On the one hand, as the policy maker wishes to avoid sudden speculative attacks, she will therefore be prone to abandoning the fixed exchange rate regime just before suffering them. On the other hand, speculators would like to be capable of anticipating the policy maker’s strategy, purchasing foreign currency in advance, which means that the policy maker and speculators play a preemption game among themselves. To avoid a speculative attack, the policy maker must play a mixed strategy so as to introduce uncertainty regarding the precise timing of the regime change. Thus, the timing of the move to a floating exchange rate regime would be a policy decision, and speculative attacks could not be predictable in the presence of an optimizing policy maker.

Assessing to what extent the mass of speculators in the economy affects the policy maker’s decision concerning relinquishing the exchange rate peg, independently of any real shocks to the economy, is the purpose of this paper, which explains currency crises as a by-product of the coordination device among speculators, regardless of economic policy mismanagement or unsound macroeconomic fundamentals. The paper introduces signalling regarding the policy maker’s decision rule. The receiver (policy maker) uses the signal (quantity of buy orders) as a device to switch between abandoning and not abandoning the peg, thus leading up to multiple equilibria in the signalling game. The basic setup of the model considered in this paper, in which the policy maker is not able to infer if the strength of the mass of speculators in the economy is sufficient for causing the collapse of the peg, is based in a simplified version of Chamley’s (2003) paper.

The paper is organized as follows. The next section sets the model up.
Section 3 in addition to presenting the equilibrium definition also characterizes equilibria in the presence of speculation. The final section concludes.

2 The Model

Following Chamley (2003), there is a finite number of periods $T + 1$, as well as a continuum of speculators, whose mass $\theta \in \{\theta_0, \theta_1\}$ is the unknown state of the economy. Let $\zeta_t$ and $\mu_{t-1}$ denote, respectively, the fraction of speculators who place a buy order for the foreign currency in period $t$ and the policy maker’s probability of the high state $\theta_1$.

The interplay between speculators and an optimizing policy maker is formalized as a dynamic Bayesian game, in which the timing of events is as follows. Nature makes the first move, choosing the value of $\theta$ that determines the state of the economy. By hypothesis, at the onset of the first period all agents own only the domestic currency, which means that if the policy maker moves first, a speculative attack will never happen at the beginning of the game. Since the policy maker reacts to speculative attacks preemptively, there is no reason for having the policy maker moving first in that she decides to abandon the peg just if she foresees that a speculative attack is oncoming. Hence speculators, who must decide whether or not to buy foreign currency in advance, move first followed by the policy maker. By observing the belief at the beginning of period $t$, as well as the fraction of speculators who place a buy order for the foreign currency and the level of foreign reserves at period $t$, the policy maker must decide whether or not to abandon the exchange rate peg.

The exchange rate is allowed to float within a predetermined band of fluctuation. If the price of the foreign currency (i.e., the exchange rate) is lower than or equal to some threshold value $1 + \gamma$, with $\gamma > 0$, then there is no devaluation in period $t$. On the other hand, if the total demand for
the foreign currency happens to be greater than some critical level of the demand, there will be a devaluation in period \( t \) and the price of the foreign currency will be equal to \( 1 + A \), with \( A > \gamma \).

2.1 The Dynamic Economy

2.1.1 The speculator’s decision problem

There exist two assets in the economy, a safe asset, also called the domestic currency, and a risky asset, also called the foreign currency. The safe asset’s return is given by 1, while the return of the risky asset is given by \( \tau_t (A - \gamma) + (1 - \tau_t) (S - \gamma) \), where \( \tau_t \) is the decision rule of the policy maker (or, equivalently, the probability of devaluation), \( A - \gamma \) is the devaluation premium, \( S \) is the spot exchange rate, and \( S - \gamma \) is the devaluation discount, with \( S < \gamma \).

The expected payoff of a representative speculator is then given by

\[
U_t = (1 - \zeta_t) u(1) + \zeta_t u [\tau_t (A - \gamma) + (1 - \tau_t) (S - \gamma)]
\]

where \( u \) is a Bernoulli utility function, continuous and strictly concave, with \( u' > 0 \) and \( u'' < 0 \). Notice that \( \zeta_t \), which describes the strategy of a domestic currency holder who places a buy order for the foreign asset, can be accordingly considered as being the probability the speculator will attack the peg.

The speculator chooses the strategy in period \( t \), \( \zeta_t \), so as to solve the following dynamic programming problem

\[
v_t(\mu_{t-1}, R_t) = \max_{\zeta_t} \{ U_t + \beta E_t [v_{t+1}(\mu_t, R_{t+1})] \}.
\]
This problem gives a policy function of the form $\zeta_t(\mu_{t-1}, R_t)$.

Since the total demand for the foreign currency by the speculators in period $t$, $\zeta_t\theta$, should affect the Central Bank’s foreign reserves level, reserves must evolve over time according to the following updating rule

$$R_t = R_{t-1} - \zeta_t\theta$$

where $R < R \leq \bar{R}$, with $R > 0$ and $\bar{R} < \infty$ being, respectively, the lowest and highest thresholds of the Central Bank’s foreign reserves level.

### 2.1.2 The policy maker’s decision problem

It is assumed that the objective function for the policy maker is similar to that proposed by Pastine (2002). On the one hand, the policy maker dislikes speculative attacks. On the other, she prefers to maintain the fixed exchange rate regime. According to Pastine, movements in reserves can be considered as a proxy for the real costs of speculative attacks. The policy maker must decide in each period whether or not to abandon the fixed exchange rate. If she decides not to abandon the peg, she will earn a payoff of $R_t$. Otherwise, she will earn $R_t - \Lambda$, where $\Lambda > 0$.

Let the state variable $R_t \in \mathcal{R} \subset \mathbb{R}^+$ and the control variable $\tau_t \in \Gamma(R_t) = [0, 1]$, where $\tau_t$ is the abandonment rule for the policy maker. If $\tau_t = 1$ the peg is abandoned, whereas $\tau_t = 0$ points to the maintenance of the peg. Under these conditions, the utility function is given by

$$\nu(R_t, \tau_t) = \tau_t V(R_t - \Lambda) + (1 - \tau_t)V(R_t),$$

where $V : \mathcal{R} \rightarrow \mathbb{R}$ with $V' > 0$ and $V'' \leq 0$.

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2 The strategy in period $t$, $\zeta_t$, should not depend on $\tau_t$, since the speculator does not know the policy maker’s abandonment rule, but rather just observes the realizations of the rule.
Since reserves depend on $\zeta_t$, which, in turn, is itself a function of $\mu_{t-1}$, the utility function must also depend on the state variables $\zeta_t$ and $\mu_{t-1}$ through their effects on $R_t$.

The policy maker chooses the abandonment rule $\tau_t$ so as to solve the following dynamic programming problem

$$W_t(\mu_{t-1}, R_t, \zeta_t) = \max_{\tau_t} \{ \nu(R_t, \tau_t) + \beta E_t [W_{t+1}(\mu_t, R_{t+1}, \zeta_{t+1})] \}$$

where $W(\mu_{t-1}, R_t, \zeta_t)$ is the value function for the policy maker. This problem provides a policy function of the form $\tau_t(\mu_{t-1}, R_t, \zeta_t)$.

## 3 Equilibrium

### 3.1 Equilibrium Definition

Speculators can choose between two actions, either attack the currency or refrain from attacking. The policy maker also has two available actions, which are merely a noncontingent choice of either abandoning or not abandoning the exchange rate peg.

Let the updating rule for beliefs, $\mu_t = M(\mu_{t-1}, \tau_t)$, be defined by the following Bayesian formula

$$\mu_t = \frac{\mu_{t-1} \tau_t}{\mu_{t-1} \tau_t + (1 - \mu_{t-1}) (1 - \tau_t)}.$$  \hspace{1cm} (5)

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3 The updating rule for beliefs, $\mu_t$, must be a function of the abandonment rule $\tau_t$. Since the abandonment rule is itself a function of the state variables $\mu_{t-1}$, $R_t$ and $\zeta_t$, so is the updating rule. The focus is on a Markov equilibrium in which strategies and beliefs depend only on the state variables.
A perfect Bayesian equilibrium for this economy consists of a set of functions $\tau_t$, $\zeta_t$, and $\mu_t$, such that (i) given the updating rule and speculators’ strategy, the abandonment rule, $\tau_t$, solves the policy maker’s dynamic programming problem given by (4); (ii) the strategy in period $t$, $\zeta_t$, solves the speculators’ dynamic programming problem given by (2); and (iii) the updating rule for beliefs, $\mu_t$, satisfies Bayes’ rule whenever possible.

The subgame in period $t$ should depend only on the state variables $\mu_{t-1}$ and $R_t$. Just symmetric equilibria will be considered.

Inasmuch as equilibrium is concerned, the updating rule for beliefs can be written as

$$M (\mu_{t-1}, \tau_t) = \begin{cases} 1 & \text{if } \tau_t = 1 \\ 0 & \text{if } \tau_t = 0 \\ \mu_{t-1} & \text{if } \tau_t = 0 \text{ and } \zeta_t = 0 \end{cases}.$$ (6)

The focus here is on a perfect Bayesian equilibrium in which the policy maker regards the fraction of speculators who place a buy order for the foreign currency in period $t$, $\zeta_t$, as a signal of the mass of speculators in the economy, $\theta$, which means that the policy maker will be prone to relinquishing the peg provided that the quantity of buy orders is sufficiently large, the magnitude of which conveys a signal about the state of the economy.

Thus, the policy maker will opt to relinquish the exchange rate peg if and only if the fraction of speculators who place a buy order for the foreign currency in period $t$, $\zeta_t$, is greater than the cutoff point $\zeta_t^*$. It then follows that the policy maker’s strategy can be written as

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4 This finding is in accordance with Chamley’s (2003 p.607) model, according to which "a devaluation takes place when the demand is higher than the critical mass."

5 Pastine (2002 p.216) argues that the policy maker "cannot follow a predictable pure strategy, since such a strategy would result in a speculative attack."
\[ \tau_t(\zeta_t) = \begin{cases} 0 \text{ w.p. } q & \text{if } \zeta_t \leq \zeta_t^* \\ 1 \text{ w.p. } (1 - q) & \text{if } \zeta_t > \zeta_t^* \end{cases}. \] (7)

The policy maker plays a mixed strategy and so randomizes between abandoning and not abandoning the peg in order to lead up to equilibrium uncertainty.

Likewise, the policy maker’s belief is

\[ \mu_t(\theta_1/\zeta_t) = \begin{cases} 0 & \text{if } \zeta_t \leq \zeta_t^* \\ 1 & \text{if } \zeta_t > \zeta_t^* \end{cases} \] (8)

where \( \theta = \theta_1 \) corresponds to the high state.

As for the speculator’s strategy, it can accordingly be written as

\[ \zeta_t \left( \mu_{t-1}, R_t \right) = \begin{cases} > 0 & \text{if } \tau_t = 1 \\ = 0 & \text{if } \tau_t = 0 \end{cases}. \] (9)

Speculators have no incentive to attack the currency if the policy maker is not to relinquish the peg, inasmuch as in this case the payoff from attacking will be lower than that of not attacking.

### 3.2 Equilibrium Determination

There exists equilibrium without speculation (\( \zeta_t = 0 \)) when \( \mu_t = 0 \), which, in turn, implies that \( \tau_t = 0 \).

If it should happen to be the case that \( \zeta_t = 0 \), then there are no speculative attacks and the game degenerates into Pastine’s complete information
case. Intuitively, this claim can be warranted since speculators are strictly risk averse. Once beliefs are such that the state of the economy is the lowest one, the optimal equilibrium strategy for the policy maker will be not to abandon the peg and play accordingly $\tau_t = 0$. Were speculators to engage in a speculative attack in some $t < T$ and a devaluation does not occur insofar as the policy maker does not relinquish the peg, their payoff would be $(S - \gamma) < 1$, since after a failed attack the spot exchange rate will accordingly be lower than the upper limit of the exchange rate band of fluctuation. The cost of such a failed attack against the peg is given by the devaluation discount $S$. Thus, speculators would have an incentive to deviate in period $t + 1$, since if they do not attack the peg and accordingly set $\zeta_t = 0$, they will earn 1, which means that $\zeta_t > 0$ cannot be an equilibrium strategy under these conditions. Since speculators are strictly risk averse and so attack the peg if and only if their expected payoff is higher than the cost of attack, they will never again buy foreign currency. It is now useful to sum this intuitive explanation up by means of the following proposition.

**Proposition 1** Once $\zeta_t = 0$ in some $t \leq T$, it stays at 0 forever.

**Proof.** By backward induction. Without loss of generality, it is assumed the constant relative risk aversion (CRRA) class of utility functions with $\rho = 1$. Consider a two-period game ($T = 1$). From (2), it then follows that in period $T + 1$

$$v_2(\mu_1, R_2) = \max_{\zeta_2} U_2(\zeta_2, \tau_2).$$

If $\tau_2 = 0$ then $U_2(\zeta_2, 0) = \zeta_2 \log (S - \gamma)$, which implies that $\zeta_2 = 0$ since $(S - \gamma) < 1$.

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6 Chamley (2003 p. 609) assumes that "if in period $t$ there is no equilibrium strategy with $\zeta_t > 0$ (no new order comes in), then the speculative attacks stop completely." Chamley (2003 p. 609, footnote 11) also conjectures that "there may be a level of belief such that no speculator buys but asset holders do not sell."
Likewise, in the first period of the game

\[ v_1(\mu_0, R_1) = \max_{\zeta_1} \{U_1(\zeta_1, \tau_1) + \beta E_1[v_2(\mu_1, R_2)]\}. \]

If \( \tau_1 = \tau_2 = 0 \) then

\[ v_1(\mu_0, R_1) = \max_{\zeta_1} \zeta_1 \log (S - \gamma) \]

since \( v_2(\mu_1, R_2) = 0 \), which implies that \( \zeta_1 = 0 \).

Thus \( \zeta_1 = \zeta_2 = 0 \).

It is straightforward to generalize this result for any \( T > 1 \).  

There also exists an equilibrium with a speculative attack \((\zeta_t > 0)\) when \( \mu_t = 1 \), which accordingly implies that \( \tau_t = 1 \). The following proposition states the main result of the paper.

**Proposition 2** There exists \( \zeta_t^* \) such that the policy maker relinquishes the currency peg if and only if \( \zeta_t > \zeta_t^* \).

**Proof.** Simple application of Tarsky’s Fixed Point Theorem.

Since \( \zeta(\mu, R) \) is a nondecreasing function, that is, \( \zeta(\mu', R') \geq \zeta(\mu, R) \) whenever \( \mu' \geq \mu, R' \geq R \), then \( \zeta(,. .) \) has a fixed point; that is, there exist \( \mu \) and \( R \) such that \( \mu, R = \zeta(\mu, R) \).\(^7\)

Since function \( \zeta(,. .) \) is increasing, there might also be multiple solutions, due to *strategic complementarity* between the policy maker’s belief about \( \theta \) and the strategy that is actually chosen by the speculator.\(^8\)

Notice that the statement that the policy maker relinquishes the currency peg if and only if \( \zeta_t > \zeta_t^* \) is equivalent to stating that the policy maker decides

\(^7\) As regards the application of Tarsky’s Fixed Point Theorem, it should be noted that \( R \) is dispensable, since \( \zeta \) can solely be written as a function of \( \mu \).

\(^8\) It is always possible to construct sunspot equilibria, in the same fashion as suggested by Jeanne and Masson (2000).
to relinquish the exchange rate peg if and only if $\mu > \mu^*$ and $R > R^*$. Since $\mu$ is the policy maker’s probability of the high state $\theta_1$, if the policy maker believes that the mass of speculators is sufficiently strong in order to bring about a successful attack against the currency, her optimal equilibrium strategy will be to relinquish the peg just before suffering the speculative attack. Furthermore, inasmuch as the policy maker considered in this model is a rational and forward-looking one, she will not wait for the complete depletion of reserves until they reach their lowest threshold level and will accordingly decide to abandon the peg while foreign reserves still belong to a comfortable range, which means that in the presence of a rational and forward-looking policy maker after a speculative attack reserves are higher than they would be in the absence of a maximizing policy maker. Thus, the policy maker will be willing to relinquish the exchange rate peg provided that foreign reserves at Central Bank, $R$, are greater than the cutoff point $R^*$.

4 Concluding Remarks

This paper has addressed the questions of incomplete information and learning, by modelling the dynamic strategic interaction between speculators and a rational and forward-looking policy maker, in an environment in which the mass of speculators is the unknown state of the economy. The policy maker does not know if the mass of speculators in the economy is sufficiently large in order to trigger a successful speculative attack against the currency. The fraction of speculators who place a buy order for the foreign currency in the market signals to the policy maker the mass of speculators in the economy.

There always exists one equilibrium with a speculative attack, in which the policy maker opts to relinquish the currency peg if and only if the fraction of speculators who place a buy order for the foreign currency is greater than the critical mass. But there might exist multiple solutions. Although the
policy maker is rational and forward-looking, the precise timing of the move to a floating regime cannot be realized due to the fact that the mass of speculators in the economy is not known by the policy maker. The exogenous uncertainty regarding the mass of potential speculators in the economy leads up to equilibrium indeterminacy. There also exists an equilibrium without a speculative attack.

The paper provides a model in which currency crises are triggered by the endogenous coordination of speculators, regardless of any real shocks to the economy, economic policy mismanagement, or even unsound macroeconomic fundamentals.

Possible extensions for future research are outlined in Appendices A and B.
References


Appendix A: Uncertainty among speculators

The model settled in the previous sections could be extended in the following way. It is now deemed that there is uncertainty among speculators in the sense that speculators do not know their own mass as well. This is the only modification concerning the basic setup of the model developed in the prior sections. With strong strategic complementarity between agents’ actions, a unique equilibrium arises.

Speculators decide whether or not to invest by taking into account the previous actions of those who have chosen before them. Since they do not know their own mass as well, they take heed of a set of predecessors who have taken their decisions before them in the previous period.

History up to period \( t \) is given by

\[
h : \{ (\zeta_1, \tau_1, \lambda_0), \ldots, (\zeta_t, \tau_t, \lambda_{t-1}) \}
\]  

(10)

where \( \lambda_{t-1}(\mu_{t-1}) \) is the fraction of speculators who have invested by the end of period \( t-1 \) and so hold the foreign currency at the beginning of period \( t \), and \( \mu_{t-1} \) is the agents’ probability of the high state \( \theta_1 \).

A trading strategy can be defined as a function of the history of the economy as well as the policy maker’s abandonment rule, \( \tau_t \), and the speculators’ strategy, \( \zeta_t \), as in

\[
\phi : \{ h \} \times [0, 1] \times [0, 1] \rightarrow [0, 1].
\]  

(11)

In what follows, a trigger strategy can be defined as
\[
\phi(h(\zeta(\mu), \tau(\mu), \lambda(\mu)), \tau(\mu), \zeta(\mu)) = \begin{cases} 
1 \text{ w.p. } p & \text{if } \zeta > \zeta^* \text{ and } \tau = 1 \\
0 \text{ w.p. } (1 - p) & \text{if } \zeta \leq \zeta^* \text{ and } \tau = 0
\end{cases}
\]

which has the property that if no speculator places a buy order for the foreign currency in a single period, then there exists no further speculation in the market thereafter.

**Proposition 3** There exists a Markov perfect equilibrium (MPE) that can be summarized by the profile of strategies \( \phi \) that are a perfect equilibrium and are measurable with respect to the payoff-relevant history \( (H_t(h_t) = H_t(\tilde{h}_t) \Rightarrow \phi_t(h_t) = \phi_t(\tilde{h}_t)) \), where \( \{H_t(h_t)\}_{t=0,...,T} \) is a summary or partition of the history of the economy.

**Proof.** Trivial application of the payoff-relevant history concept.\(^9\) By backward induction, the subgame in each period selects a Nash equilibrium that is the same for all histories \( \{h_t\}_{t=0,...,T} \). □

Combining the Markov restriction with the iterated elimination of strictly dominated strategies, then a unique Markov perfect equilibrium arises. It suffices to notice that the elimination of strictly dominated strategies reduces the set of state variables.

Thus, with strong strategic complementarity between agents’ actions, the *global games* unique result arises.

\(^9\) For further details see Fudenberg and Tirole (1991, ch. 13).
Appendix B: Empirical estimation approach

In order to assess to what extent the mass of speculators in the economy is a determining factor concerning the policy maker’s decision of relinquishing the currency peg, a state-space representation is assumed for an MSI-VAR model in which the Markov chain governing the state vector follows a VAR (1) process, such as

\[
\begin{bmatrix}
\theta_{t+1} \\
\mu_{t+1} \\
\zeta_{t+1} \theta
\end{bmatrix}
= \begin{bmatrix}
\phi_\theta & 0 & 0 \\
0 & \phi_\mu & 0 \\
0 & 0 & \phi_\zeta \theta
\end{bmatrix}
\begin{bmatrix}
\theta_t \\
\mu_t \\
\zeta_t \theta
\end{bmatrix}
+ D \zeta_t
+ \begin{bmatrix}
v_{\theta_{t+1}} \\
v_{\mu_{t+1}} \\
v_{\zeta_{t+1} \theta}
\end{bmatrix}
\]  

and Measurement Equation

\[
[R_t] = [\alpha_1] + \begin{bmatrix}
0 & 0 & \gamma
\end{bmatrix}
\begin{bmatrix}
\theta_t \\
\mu_t \\
\zeta_t \theta
\end{bmatrix}
+ [0] \zeta_t
\]

where

\[\theta \in \{\theta_0, \theta_1\} = \text{mass of speculators};\]

\[\mu_t = \text{policy maker’s probability of the high state } \theta_1;\]
\( R_t = \) foreign reserves; and

\( \zeta_t = \) variable of control, fraction of speculators who place a buy order for the foreign currency (speculator’s strategy).

The vector of observed variables can be written as

\[
R_t = \alpha_1 + \gamma \zeta_t \theta. \tag{15}
\]

The evolution of the decision rule of the policy maker (probability of devaluation), \( \tau_t \in [0, 1] \), is dependent upon \( \psi_t = (\mu_{t-1}, R_t, \zeta_t) \), such as

\[
\Pr[\tau_t = 0|\psi_t] = q = \frac{\exp\left(q_0 + \psi_t' q_1\right)}{1 + \exp\left(q_0 + \psi_t' q_1\right)}, \tag{16}
\]

\[
\Pr[\tau_t = 1|\psi_t] = (1 - q) = 1 - \frac{\exp\left(q_0 + \psi_t' q_1\right)}{1 + \exp\left(q_0 + \psi_t' q_1\right)}. \tag{17}
\]