

# Practical Volatility and Correlation Modeling for Financial Market Risk Management

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Andersen, Bollerslev, Christoffersen, and Diebold (henceforth ABCD) provide a comprehensive overview of financial risk management from the point of view of both Wall Street and the Ivory Tower. Most usefully, ABCD discuss a number of recent developments in the econometrics of time varying risk that hold vast promise for risk management applications: the dynamic conditional correlation model of Engle (2002) which permits large-scale, flexible modeling of conditional covariance matrices; the use of high-frequency data to measure realized variances and covariances that has been developed largely by the authors; and the modeling of the full distribution of conditional returns. In this discussion I will just offer a couple of comments and extensions to ABCD's very well organized survey.

## Unconditional vs Conditional Risk

ABCD discuss extensively the pros and cons of both unconditional and conditional (dynamic) measures of risk. There is however an additional source of risk dynamics that is ignored in the paper and that, in fact, has not been studied much in the literature. Most financial assets are managed over time and it is therefore more important to study the risks of dynamic investment strategies rather than the risks of static portfolios. Especially for supervision and regulation purposes, it matters more to forecast the risk of a portfolio taking into account the likely variation in its weights than to forecast the risk of the current positions that are unlikely to remain in place for long.

Assume that there exist some state variables that forecast both risk and return. A trader that adjusts the portfolio according to those state variables, for instance to maximize the conditional Sharpe ratio, will produce a portfolio with time-varying risk. Many authors have shown that the level of interest rates, the term spread, and the default spread have forecasting power for both first and second moments of returns of stocks and bonds. Brandt and Santa-Clara (2005) show that the optimal asset allocation for a mean-variance investor that recognizes the forecasting power of these state variables displays considerable time variation in portfolio weights and conditional moments.

As another example, investment strategies are typically conditioned on the level of risk in the markets. Either formally, through VaR constraints, or informally, according to the trader's feelings, the level of exposure is adjusted when risks change. Consider a trader with a VaR limit that manages the exposure of the portfolio to always be at that limit. When market risk is high, the exposure is reduced, and when risk is low, the exposure is increased.

Interestingly, the result of this dynamic strategy is a series of returns that have constant conditional VaR. That is, in this case, a dynamic strategy produces a series of returns with static risk.

This example explains why the realized risk of a managed portfolio may not display GARCH characteristics even though the assets in the portfolio have them. Ex ante, if the portfolio were to remain constant, its risk would be changing. Ex post, given that the portfolio changes with the ex ante risk assessment, the realized risk is not time varying. This distinction between ex ante and ex post risk of an investment strategy has been the basis of much confusion relating to the need of unconditional vs conditional risk models. It justifies the use of unconditional VaR by regulators since they care only about ex post risk. On the other hand, traders need the more sophisticated models of conditional risk to be able to manage the exposures in a timely manner.

### Modeling the Entire Distribution of Returns

ABCD explain that the common use of summary statistics such as volatility, VaR, or expected shortfall is likely to give a partial view of the true risk of a portfolio. Only the full (conditional) distribution of returns, including skewness and fat tails, will capture correctly the likelihood of different levels of losses.

Santa-Clara and Schwartz (2005) offer a simple alternative that captures the impact of the full distribution of returns on the risk of a portfolio. Their approach can be summarized briefly. The idea is that the investor (or the regulator) analyzes the distribution of returns through the lens of a utility function of returns that is concave (reflecting risk aversion). A simple example is the well-known power utility function,  $u(r) = (1 + r)^{1-\gamma}/(1 - \gamma)$ , with relative risk aversion  $\gamma$ .

Given portfolio weights  $w$ , simulate the history of portfolio returns:

$$r_{p,t+1} = \sum_{i=1}^N w_i r_{i,t+1} \quad \text{for } t = 1, \dots, T - 1$$

and evaluate the corresponding time series of realized utilities of the portfolio  $u(r_{p,t+1})$ . Then, regress the realized utilities on state variables  $z$  that condition the joint return distribution:<sup>1</sup>

$$u(r_{p,t+1}) = \phi z_t + \epsilon_{t+1} .$$

The fitted values of this regression are estimates of the conditional expected utility  $E_t[u(r_{p,t+1})]$ . At the current time  $T$ , the regression is estimated with historic data, and the fitted value  $E_T[u(r_{p,T+1})] = \hat{\phi} z_T$  is a forecast of the risk of the portfolio in the next period  $T + 1$ . Actually, a more easily interpreted measure of risk is the conditional certainty equivalent  $c_t = u^{-1}(E_t[u(r_{p,t+1})])$  which is expressed in units of returns.

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<sup>1</sup>The variables  $z$  may contain basis functions of a more fundamental set of state variables  $y$ . In this way the specification can accommodate a nonlinear relation between  $y$  and the expected utility. Also, the returns may be demeaned prior to running the regression in order to concentrate on risk and discard the effect of the average return on the investor's utility.

We can run similar regressions for the partial derivatives of the expected utility relative to portfolio weights. These derivatives can be used for risk management as they quantify how much the utility (or certainty equivalent) changes when the weight of each asset changes marginally.

Santa-Clara and Schwartz' measure of risk takes into account the full distribution of returns. The investor cares about the expected value of the utility, which in turn depends on all the moments of the distribution of the portfolio returns:

$$E_t[u(r_{p,t+1})] \approx u(E_t[r_{p,t+1}]) + u''(E_t[r_{p,t+1}])\text{Var}_t[r_{p,t+1}]/2 + u'''(E_t[r_{p,t+1}])\text{Skew}_t[r_{p,t+1}]/6 + \dots$$

which depend implicitly on the full joint distribution of the assets' returns. We have therefore a measure of risk that combines all the features of the distribution of returns weighted in an optimal manner according to the risk preferences of the investor.

Finally, this approach can easily accommodate dynamic investment strategies. Simply model the portfolio weights as a function of state variables  $x_t$  (which may or may not be different from  $z_t$ ):

$$r_{p,t+1} = \sum_{i=1}^{N_t} w_{i,t} r_{i,t+1} = \sum_{i=1}^N (\theta x_t) r_{i,t+1},$$

compute the realized utilities, and perform the regression above. Going a step further, the coefficients of the portfolio policy can be optimized to maximize the conditional expected utility of the portfolio along the lines of Brandt and Santa-Clara (2005) and Brandt, Santa-Clara, and Valkanov (2005):

$$\max_{\theta} \frac{1}{T} \sum_{t=1}^T u(r_{p,t+1}) = \frac{1}{T} \sum_{t=1}^T u \left( \sum_{i=1}^N (\theta x_t) r_{i,t+1} \right)$$

## Conclusion

The econometrics of risk is an exciting area right now. ABCD's paper is a precious guide to recent developments and points interesting directions for future research.

## References

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